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Archive - JEE Main & Advanced

Quadratic Equations	Class - XI | Mathematics

JEE Main 2021

1. Let p and q be two positive numbers such that $p+q=2$ and $p^4+q^4=272$. Then p and q are roots of the equation:

(A) $x^2-2x+136=0$ **(B)** $x^2-2x+2=0$

(C) $x^2-2x+16=0$ **(D)** $x^2-2x+8=0$

2. The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is ______ .

3. The integer 'k', for which the inequality $x^2-2(3k-1)x+8k^2-7>0$ is valid for every x in R, is :

(A) 4 **(B)** 3 **(C)** 0 **(D)** 2

4. Let α and β be the roots of $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$ for $n \ge 1$, then the value of $\frac{a_{10}-2a_8}{3a_9}$ is:

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1

5. Let α and β be two real numbers such that $\alpha+\beta=1$ and $\alpha\beta=-1$. Let $p_n=(\alpha)^n+(\beta)^n$, $p_{n-1}=11$ and $p_{n+1}=29$ for some integer $n \ge 1$. Then, the value of p_n^2 is __________ .

6. Let $f:[-1,1]\to R$ be defined as $f(x)=ax^2+bx+c$ for all $x\in[-1,1]$, where $a,b,c\in R$ such that $f(-1)=2$, $f'(-1)=1$ and for $x\in(-1,1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x)\le\alpha$, $x\in[-1,1]$, then the least value of α is equal to _______.

7. The value of $3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\dots\dots\infty}}}}$ is equal to:

(A) $2+\sqrt{3}$ **(B)** $3+2\sqrt{3}$

(C) $4+\sqrt{3}$ **(D)** $1.5+\sqrt{3}$

8. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\dots\dots\infty}}}}$ is :

(A) $2+\frac{4}{\sqrt{5}}\sqrt{30}$ **(B)** $2+\frac{2}{5}\sqrt{30}$ **(C)** $4+\frac{4}{\sqrt{5}}\sqrt{30}$ **(D)** $5+\frac{2}{5}\sqrt{30}$

9. If α and β are the distinct roots of the equation $x^2+(3)^{1/4}x+3^{1/2}=0$, then the value of $\alpha^{96}(\alpha^{12}-1)+\beta^{96}(\beta^{12}-1)$ is equal to ;

(A) 28×3^{25} **(B)** 52×3^{24} **(C)** 56×3^{25} **(D)** 56×3^{24}

10. The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0,\ x>0$, is _____.

11. If α, β are roots of the equation $x^2+5(\sqrt{2})x+10=0, \alpha>\beta$ and $P_n=\alpha^n-\beta^n$ for each positive integer n, then the value of $\left(\dfrac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to ___________ .

12. The number of real solutions of the equation, $x^2-|x|-12=0$ is:

(A) 1 **(B)** 2 **(C)** 4 **(D)** 3

13. If $a+b+c=1, ab+bc+ca=2$ and $abc=3$, then the value of $a^4+b^4+c^4$ is equal to _______.

14. Let α, β be two roots of the equation $x^2+(20)^{1/4}x+(5)^{1/2}=0$. Then $\alpha^8+\beta^8$ is equal to :

(A) 160 **(B)** 10 **(C)** 50 **(D)** 100

15. The number of real roots of the equation $e^{4x}-e^{3x}-4e^{2x}-e^x+1=0$ is equal to _________.

16. The number of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2+ax+b=0, \alpha^2-2$ is also a root of this equation is:

(A) 4 **(B)** 2 **(C)** 8 **(D)** 6

17. The set of all values of $k>-1$, for which the equation $(3x^2+4x+3)^2-(k+1)\ (3x^2+4x+3)\ (3x^2+4x+2)+k(3x^2+4x+2)^2=0$ has real roots, is :

(A) $[2, 3)$ **(B)** $\left(\frac{1}{2}, \frac{3}{2}\right]-\{1\}$ **(C)** $\left[-\frac{1}{2}, 1\right)$ **(D)** $\left(1, \frac{5}{2}\right]$

18. Let $\lambda \neq 0$ be in R. If α and β are roots of the equation $x^2-x+2\lambda=0$, and α and γ are the roots of the equation $3x^2-10x+27\lambda=0$, then $\dfrac{\beta\gamma}{\lambda}$ is equal to ___________.

19. The sum of all integral values of $k\ (k \neq 0)$ for which the equation $\dfrac{2}{x-1}-\dfrac{1}{x-2}=\dfrac{2}{k}$ in x has no real roots, is ______.

JEE Advanced 2021

1. For $x \in R$, the number of real roots of the equation $3x^2-4|x^2-1|+x-1=0$ is _______.

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Archive - JEE Main & Advanced

Quadratic Equations	Class - XI | Mathematics

JEE Main 2022

1. The minimum value of the sum of the squares of the roots of $x^2+(3-a)x+1=2a$ is:

(A) 4 **(B)** 5 **(C)** 6 **(D)** 8

2. Let $\alpha,\beta(\alpha>\beta)$ be the roots of the quadratic equation $x^2-x-4=0$. If $P_n=a^n-\beta^n, n\in N$, then $\dfrac{P_{15}P_{16}-P_{14}P_{16}-P_{15}^2+P_{14}P_{15}}{P_{13}P_{14}}$ is equal to ________.

3. The sum of all the real roots of the equation $(e^{2x}-4)(6e^{2x}-5e^x+1)=0$ is:

(A) $\log_e 3$ **(B)** $-\log_e 3$ **(C)** $\log_e 6$ **(D)** $-\log_e 6$

4. The number of distinct real roots of the equation $x^7-7x-2=0$ is:

(A) 5 **(B)** 7 **(C)** 1 **(D)** 3

5. If the sum of the squares of the reciprocals of the roots α *and* β of the equation $3x^2+\lambda x-1=0$ is 15, then $6(\alpha^3+\beta^3)^2$ is equal to:

(A) 18 **(B)** 24 **(C)** 36 **(D)** 96

6. Let $a,b\in R$ be such that the equation $ax^2-2bx+15=0$ has a repeated root α. If α and β are the roots of the equation $x^2-2bx+21=0$, then $\alpha^2+\beta^2$ is equal to:

(A) 37 **(B)** 58 **(C)** 68 **(D)** 92

7. The sum of the cubes of all the roots of the equation $x^4-3x^3-2x^2+3x+1=0$ is ________.

8. Let α,β be the roots of the equation $x^2-4\lambda x+5=0$ *and* α,γ be the roots of the equation $x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+7+3\lambda\sqrt{3}=0, \lambda>0$. If $\beta+\gamma=3\sqrt{2}$, then $\left(\alpha+2\beta+\gamma\right)^2$ is equal to _______.

9. The number of distinct real roots of $x^4-4x+1=0$.

(A) 4 **(B)** 2 **(C)** 1 **(D)** 0

10. If the sum of all the roots of equation $e^{2x}-11e^x-45e^{-x}+\dfrac{81}{2}=0$ is $\log_e p$, then p is equal to ______.

11. Let $f(x)$ be a quadratic polynomial such that $f(-2)+f(3)=0$. If one of the roots of $f(x)=0$ is –1, then the sum of the roots of $f(x)=0$ is equal to:

(A) $\dfrac{11}{3}$ **(B)** $\dfrac{7}{3}$ **(C)** $\dfrac{13}{3}$ **(D)** $\dfrac{14}{3}$

12. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to ______.

(A) 0 **(B)** 1 **(C)** 3 **(D)** 5

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Archive - JEE Main & Advanced

Trigonometry	Class - XI | Mathematics

JEE Main 2021

1. Let $\alpha = \max\limits_{x \in \boldsymbol{R}} \left\{8^{2\sin 3x}.4^{4\cos 3x}\right\}$ and $\beta = \min\limits_{x \in \boldsymbol{R}} \left\{8^{2\sin 3x}.4^{4\cos 3x}\right\}$

If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value c – b is equal to:

(A) 42 **(B)** 43 **(C)** 50 **(D)** 47

2. $\cos ec 18°$ is a root of the equation:

(A) $x^2 - 2x - 4 = 0$ **(B)** $4x^2 + 2x - 1 = 0$ **(C)** $x^2 - 2x + 4 = 0$ **(D)** $x^2 + 2x - 4 = 0$

3. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + ...\infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\dfrac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \dfrac{\pi}{2}\right)$ is:

(A) $\sqrt{3}$ **(B)** $\dfrac{1}{2}$ **(C)** $\dfrac{3}{2}$ **(D)** $2\sqrt{3}$

4. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their top to be complementary, then the height of the shorter pole (in meters) is:

(A) 30 **(B)** 20 **(C)** $25\sqrt{3}$ **(D)** $20\sqrt{3}$

5. All possible values of $\theta = [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

(A) $\left(0, \dfrac{\pi}{2}\right) \cup \left(\pi, \dfrac{3\pi}{2}\right)$ **(B)** $\left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right) \cup \left(\pi, \dfrac{5\pi}{4}\right) \cup \left(\dfrac{3\pi}{2}, \dfrac{7\pi}{4}\right)$

(C) $\left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right) \cup \left(\dfrac{3\pi}{2}, \dfrac{11\pi}{6}\right)$ **(D)** $\left(0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right) \cup \left(\pi, \dfrac{7\pi}{6}\right)$

6. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \dfrac{3}{2}$, then $\sin x + \cos y$ is equal to:

(A) $\dfrac{1 - \sqrt{3}}{2}$ **(B)** $\dfrac{\sqrt{3}}{2}$ **(C)** $\dfrac{1 + \sqrt{3}}{2}$ **(D)** $\dfrac{1}{2}$

7. The number integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is _______.

8. If $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \dfrac{\pi}{2}\right]$ is ______.

9. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is:

(A) $3600\sqrt{3}m$ **(B)** $1800\sqrt{3}m$ **(C)** $1200\sqrt{3}m$ **(D)** $2400\sqrt{3}m$

10. The number of solutions of the equation $x + 2\tan x = \dfrac{\pi}{2}$ in the internal $[0, 2\pi]$ is :

(A) 5 **(B)** 3 **(C)** 4 **(D)** 2

11. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is ____________ .

12. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to:

(A) $\sqrt{3}$ **(B)** $2\sqrt{3}$ **(C)** $\frac{2\sqrt{3}}{3}$ **(D)** $\frac{1}{\sqrt{3}}$

13. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8 cosec^6\alpha$ is equal to:

(A) 400 **(B)** 250 **(C)** 500 **(D)** 350

14. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to :

(A) 4 **(B)** 2 **(C)** 8 **(D)** 3

15. The value of $\cot\frac{\pi}{24}$ is:

(A) $3\sqrt{2} - \sqrt{3} - \sqrt{6}$ **(B)** $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$

(C) $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$ **(D)** $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

16. If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to :

(A) -23 **(B)** 23 **(C)** -27 **(D)** 27

17. Let $f: \boldsymbol{R} \to \boldsymbol{R}$ be defined as

$f(x+y) + f(x-y) = 2f(x)f(y)$, $f\left(\frac{1}{2}\right) = -1$. Then, the value of $\sum_{k=1}^{20} \frac{1}{\sin(k)\sin(k + f(k))}$ is equal to :

(A) $cosec^2(1)cosec(21)\sin(20)$ **(B)** $cosec^2(21)\cos(20)\cos(2)$

(C) $\sec^2(21)\sin(20)\sin(2)$ **(D)** $\sec^2(1)\sec(21)\cos(20)$

18. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to :

(A) 1 **(B)** 0 **(C)** 3 **(D)** 4

19. If n is the number of solutions of the equation $2\cos x\left(4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right) = 1$, $x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is:

(A) $\left(2, \frac{8\pi}{9}\right)$ **(B)** $\left(3, \frac{13\pi}{9}\right)$ **(C)** $\left(3, \frac{5\pi}{3}\right)$ **(D)** $\left(2, \frac{2\pi}{3}\right)$

20. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground $18m$ away from the base of the pole, then the height of the pole (in meters) is:

(A) $12\sqrt{15}$ **(B)** $12\sqrt{10}$ **(C)** $6\sqrt{10}$ **(D)** $8\sqrt{10}$

21. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to _______.

22. Two poles, AB of length a meters and CD of length $a+b(b \neq a)$ meters are erected at the same horizontal level with bases at B and D. If $BD = x$ and $\tan\underline{|ACB} = \frac{1}{2}$, then :

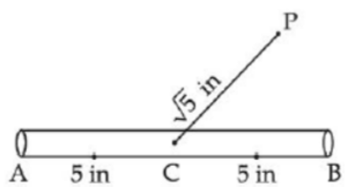
(A) $x^2 + 2(a+2b)x - b(a+b) = 0$ **(B)** $x^2 - 2ax + a(a+b) = 0$

(C) $x^2 - 2ax + b(a+b) = 0$ **(D)** $x^2 + 2(a+2b)x + a(a+b) = 0$

23. The value of $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$ is:

(A) $\frac{1}{4}$ **(B)** $\frac{1}{4\sqrt{2}}$ **(C)** $\frac{1}{8\sqrt{2}}$ **(D)** $\frac{1}{8}$

24. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC = \sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is:



(A) $\tan^{-1}\left(\frac{4}{3}\right)$ **(B)** $\tan^{-1}(1)$ **(C)** $\tan^{-1}\left(\frac{1}{2}\right)$ **(D)** $\tan^{-1}\left(\frac{3}{4}\right)$

25. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is:

(A) $-\frac{\pi}{15}$ **(B)** $-\frac{11\pi}{30}$ **(C)** $-\frac{7\pi}{30}$ **(D)** $\frac{\pi}{10}$

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Trigonometry	Class - XI | Mathematics

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1. The number of solutions of the equation $\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x,\ x\in[-3\pi, 3\pi]$ is:

(A) 8 **(B)** 5 **(C)** 6 **(D)** 7

2. The sum of absolute maximum and absolute minimum values of the function $f(x)=|2x^2+3x-2|+\sin x\cos x$ in the interval [0, 1] is:

(A) $3+\frac{\sin(1)\cos^2\left(\frac{1}{2}\right)}{2}$ **(B)** $3+\frac{1}{2}(1+2\cos(1))\sin(1)$

(C) $5+\frac{1}{2}(\sin(1)+\sin(2))$ **(D)** $2+\sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

3. Let $S=\left\{\theta\in[-\pi,\pi]-\left\{\pm\frac{\pi}{2}\right\}:\sin\theta\tan\theta+\tan\theta=\sin 2\theta\right\}$. If $T=\sum_{\theta\in S}\cos 2\theta$, then $T+n(S)$ is equal to:

(A) $7+\sqrt{3}$ **(B)** 9 **(C)** $8+\sqrt{3}$ **(D)** 10

4. The value of $2\sin(12^\circ)-\sin(72^\circ)$ is:

(A) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$ **(B)** $\frac{1-\sqrt{5}}{8}$ **(C)** $\frac{\sqrt{3}(1-\sqrt{5})}{2}$ **(D)** $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

5. The number of values of x in the interval $\left(\frac{\pi}{4},\frac{7\pi}{4}\right)$ for which $14\,\mathrm{cosec}^2 x-2\sin^2 x=21-4\cos^2 x$ holds, is________.

6. $16\sin(20^\circ)\sin(40^\circ)\sin(80^\circ)$ is equal to:

(A) $\sqrt{3}$ **(B)** $2\sqrt{3}$ **(C)** 3 **(D)** $4\sqrt{3}$

7. If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ)=\alpha-\frac{1}{16}\sin(10^\circ)$, then $16+\alpha^{-1}$ is equal to ___________.

8. $\alpha=\sin 36^\circ$ is a root of which of the following equation?

(A) $16x^4-10x^2-5=0$ **(B)** $16x^4+20x^2-5=0$

(C) $16x^4-20x^2+5=0$ **(D)** $16x^4-10x^2+5=0$

9. The value of $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$ is equal to:

(A) -1 **(B)** $-\frac{1}{2}$ **(C)** $-\frac{1}{3}$ **(D)** $-\frac{1}{4}$

10. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are:

(A) $-\frac{1}{7}$ and IV^{th} quadrant **(B)** 7 and IV^{th} quadrant

(C) -7 and IV^{th} quadrant **(D)** $\frac{1}{7}$ and I^{st} quadrant

11. Let *AB* and *PQ* be two vertical poles, 160 *m* apart from each other. Let *C* be the middle point of *B* and *Q*, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from *C* to *P* and *A*, respectively. If the height of pole *PQ* is twice the height of pole *AB*, then $\tan^2\theta$ is equal to:

(A) $\frac{3-2\sqrt{2}}{2}$ **(B)** $\frac{3+\sqrt{2}}{2}$ **(C)** $\frac{3-2\sqrt{2}}{4}$ **(D)** $\frac{3-\sqrt{2}}{4}$

12. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is:

(A) $15\sqrt{3}$ **(B)** $20\sqrt{3}$ **(C)** $20+10\sqrt{3}$ **(D)** 30

13. The number of solutions of the equation $\sin x = \cos^2 x$ in the interval (0, 10) is ______.

14. The number of elements in the set

$S = \left\{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta - 10\cos^2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\right\}$ is _________.

15. The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ in *R* is equal to _________.

16. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to:

(A) $\frac{3}{16}$ **(B)** $\frac{1}{16}$ **(C)** $\frac{1}{32}$ **(D)** $\frac{9}{32}$

17. Let a vertical tower *AB* of height 2*h* stands on a horizontal ground. Let from a point *P* on the ground a man can see upto height *h* of the tower with an angle of elevation 2α. When from *P*, he moves a distance *d* in the direction of $\overrightarrow{AP}$, he can see the top *B* of the tower with an angle of elevation α. If $d = \sqrt{7}h$, then $\tan\alpha$ is equal to:

(A) $\sqrt{5}-2$ **(B)** $\sqrt{3}-1$ **(C)** $\sqrt{7}-2$ **(D)** $\sqrt{7}-\sqrt{3}$

18. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is:

(A) 4 **(B)** 6 **(C)** 8 **(D)** 12

19. A tower *PQ* stands on a horizontal ground with base *Q* on the ground. The point *R* divides the tower in two parts such that $QR = 15m$. If from a point *A* on the ground the angle of elevation of *R* is 60° and the part *PR* of the tower subtends an angle of 15° at *A*, then the height of the tower is:

(A) $5(2\sqrt{3}+3)\ m$ **(B)** $5(\sqrt{3}+3)\ m$

(C) $10(\sqrt{3}+1)\ m$ **(D)** $10(2\sqrt{3}+1)\ m$

20. Let $S = \left[-\pi, \frac{\pi}{2}\right) - \left\{-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}\right\}$. Then the number of elements in the set $A = \left\{\theta \in S : \tan\theta\left(1+\sqrt{5}\tan(2\theta)\right) = \sqrt{5} - \tan(2\theta)\right\}$ is ____________.

21. A horizontal park is in the shape of a triangle *OAB* with *AB* = 16. A vertical lamps post *OP* is erected at the point *O* such that $\angle PAO = \angle PBO = 15^o$ and $\angle PCO = 45^o$, where *C* is the midpoint of *AB*. Then $(OP)^2$ is equal to:

(A) $\frac{32}{\sqrt{3}}\left(\sqrt{3}-1\right)$ **(B)** $\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$ **(C)** $\frac{16}{\sqrt{3}}\left(\sqrt{3}-1\right)$ **(D)** $\frac{16}{\sqrt{3}}\left(2-\sqrt{3}\right)$

22. The angle of elevation of the top of a tower from a point *A* due north of it is α and from a point *B* at a distance of 9 units due west of *A* is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point *B* from the tower is 15 units, then $\cot\alpha$ is equal to:

(A) $\frac{6}{5}$ **(B)** $\frac{9}{5}$ **(C)** $\frac{4}{3}$ **(D)** $\frac{7}{3}$

23. Let $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$. Then, the sum of roots of all the equations $x^2 - 2\left(\tan^2\theta + \cot^2\theta\right)x + 6\sin^2\theta = 0$, $\theta \in S$, is _________.

24. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to ______.

25. The angle of elevation of the top *P* of a vertical tower *PQ* of height 10 from a point *A* on the horizontal ground is 45°. Let *R* be a point on *AQ* and from a point *B*, vertically above *R*, the angle of elevation of *P* is 60°. If $\angle BAQ = 30°, AB = d$ and the area of the trapezium *PQRB* is α, then the ordered pair (d, α) is:

(A) $\left(10\left(\sqrt{3}-1\right), 25\right)$ **(B)** $\left(10\left(\sqrt{3}-1\right), \frac{25}{2}\right)$

(C) $\left(10\left(\sqrt{3}+1\right), 25\right)$ **(D)** $\left(10\left(\sqrt{3}+1\right), \frac{25}{2}\right)$

26. Let $S = \left\{\theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$. Then

(A) $S = \left\{\frac{\pi}{12}\right\}$ **(B)** $S = \left\{\frac{2\pi}{3}\right\}$ **(C)** $\sum_{\theta \in S} \theta = \frac{\pi}{2}$ **(D)** $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

27. Let $S = \left\{\theta \varepsilon [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\right\}$. Then

$n(S) + \sum_{\theta \varepsilon S}\left(\sec\left(\frac{\pi}{4} + 2\theta\right) cosec\left(\frac{\pi}{4} + 2\theta\right)\right)$ is equal to:

(A) 0 **(B)** –2 **(C)** –4 **(D)** 12

Archive - JEE Main & Advanced

Sequence and Series	Class - XI | Mathematics

JEE Main 2021

1. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is:

(A) $\frac{71}{256}$ **(B)** $-\frac{71}{256}$ **(C)** $\frac{69}{256}$ **(D)** $-\frac{69}{256}$

2. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is ____________.

3. If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n}\theta, y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \sin^{2n}\phi$ then :

(A) $xy - z = (x + y)z$ **(B)** $xy + yz + zx = z$

(C) $xyz = 4$ **(D)** $xy + z = (x + y)z$

4. Let $A_1, A_2, A_3, ...$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If length of A_1 is $12cm$, then the smallest value of n for which area of A_n is less than one, is_______.

5. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to:

(A) $2\sqrt{a}$ **(B)** $2a$ **(C)** $a + 1$ **(D)** $a + \frac{1}{a}$

6. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to :

(A) 30 **(B)** 26 **(C)** 35 **(D)** 32

7. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + ...$ is equal to :

(A) $\frac{11}{4}$ **(B)** $\frac{9}{4}$ **(C)** $\frac{15}{4}$ **(D)** $\frac{13}{4}$

8. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to:

(A) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$ **(B)** $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

(C) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$ **(D)** $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

9. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence $-16, 8, -4, 2, ...$ satisfy the equation $4x^2 - 9x + 5 = 0$, then $p + q$ is equal to ________ .

10. If 1, $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of determinant $\begin{vmatrix} 2\left(x - \frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to :

11. If α, β are natural numbers such that $100^\alpha - 199\beta = (100)(100) + (99)(101) + (98)(102) + ... + (1)(199)$, then the slope of the line passing through (α, β) and origin is:

(A) 540 **(B)** 510 **(C)** 550 **(D)** 530

12. $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + ... + \frac{1}{(201)^2-1}$ is equal to:

(A) $\frac{99}{400}$ **(B)** $\frac{25}{101}$ **(C)** $\frac{101}{404}$ **(D)** $\frac{101}{408}$

13. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P., where $a, b > 0$. Then $72(a+b)$ is equal to ________.

14. Let $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + ...$ upto n-terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to ________ .

15. Let S_1 be the sum of first $2n$ terms of an arithmetic progression. Let S_2 be the sum of first $4n$ terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first $6n$ terms of the arithmetic progression is equal to:

(A) 1000 **(B)** 3000 **(C)** 5000 **(D)** 7000

16. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to _______.

17. If sum of the first 21 terms of the series $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + ...$, where $x > 0$ is 504, then x is equal to:

(A) 87 **(B)** 7 **(C)** 243 **(D)** 9

18. For $k \in N$, let $\frac{1}{\alpha(\alpha+1)(\alpha+2).....(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$, where $\alpha > 0$. Then the value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to ________ .

19. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \ge 1$. Then the value of $47\sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$ is equal to ___________ .

20. The sum of all the elements in the set $\{n \in \{1, 2, ..., 100\} \mid$ H.C.F. of n and 2040 is 1$\}$ is equal to _____.

21. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is:

(A) 8 **(B)** 4 **(C)** 6 **(D)** 2

22. If the value of $\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+.......\text{upto}\infty\right)^{\log_{(0.25)}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+......\text{upto}\infty\right)}$ is l, then l^2 is equal to ________ .

23. If $[x]$ be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to:

(A) -2 **(B)** 0 **(C)** 2 **(D)** 4

24. If $\log_3 2, \log_3(2^x-5), \log_3\left(2^x-\frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to _______.

25. Let $S_n = 1\cdot(n-1)+2\cdot(n-2)+3\cdot(n-3)+\ldots+(n-1)\cdot 1, n\ge 4$. The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!}-\frac{1}{(n-2)!}\right)$ is equal to:

(A) $\frac{e}{6}$ **(B)** $\frac{e-1}{3}$ **(C)** $\frac{e}{3}$ **(D)** $\frac{e-2}{6}$

26. Let $a_1, a_2, \ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20}\frac{1}{a_n a_{n+1}}=\frac{4}{9}$. If the sum of this AP is 189, then $a_6 a_{16}$ is equal to:

(A) 48 **(B)** 72 **(C)** 57 **(D)** 36

27. The mean of 10 numbers $7\times 8, 10\times 10, 13\times 12, 16\times 14, \ldots$ is ___________ .

28. The sum of 10 terms of the series $\frac{3}{1^2\times 2^2}+\frac{5}{2^2\times 3^2}+\frac{7}{3^2\times 4^2}+\ldots$ is:

(A) $\frac{143}{144}$ **(B)** $\frac{99}{100}$ **(C)** 1 **(D)** $\frac{120}{121}$

29. Three numbers are in an increasing geometric progression with common ration r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is 3 r^2, then r^2-d is equal to:

(A) $7-7\sqrt{3}$ **(B)** $7+\sqrt{3}$ **(C)** $7-\sqrt{3}$ **(D)** $7+3\sqrt{3}$

30. If for $x, y \in R, x>0, y=\log_{10}x+\log_{10}x^{1/3}+\log_{10}x^{1/9}+\ldots$ upto ∞ terms and $\frac{2+4+6+\ldots+2y}{3+6+9+\ldots+3y}=\frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to:

(A) $(10^6, 6)$ **(B)** $(10^2, 3)$ **(C)** $(10^6, 9)$ **(D)** $(10^4, 6)$

31. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is ________.

32. Let $a_1, a_2, \ldots\ldots, a_{10}$ be an AP with common difference – 3 and $b_1, b_2, \ldots, b_{10}$ be a GP with common ratio 2. Let $c_k = a_k + b_k, k = 1, 2, \ldots, 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to __________.

33. If ${}^1P_1 + 2\cdot{}^2P_2 + 3\cdot{}^3P_3 + \ldots. + 15\cdot{}^{15}P_{15} = {}^qP_r - s, 0\le s\le 1$. then ${}^{q+s}C_{r-s}$ is equal to _______.

34. If the sum of an infinite GP $a, ar, ar^2, ar^3, \ldots$ is 15 and the sum of the squares of its each term is 150, then the sum of $ar^2, ar^4, ar^6, \ldots$ is:

(A) $\frac{1}{2}$ **(B)** $\frac{9}{2}$ **(C)** $\frac{5}{2}$ **(D)** $\frac{25}{2}$

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Sequence and Series	Class - XI | Mathematics

JEE Main 2022

1. The remainder on dividing $1+3+3^2+3^3+.....+3^{2021}$ by 50 is _______.

2. If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192, \sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to:

(A) 48 **(B)** 96 **(C)** 92 **(D)** 104

3. The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, ...$ is equal to _______.

4. If $a_1(>0), a_2, a_3, a_4, a_5$ are in a G.P., $a_2+a_4 = 2a_3+1$ and $3a_2+a_3 = 2a_4$, then $a_2+a_4+2a_5$ is equal to _______.

5. Let $S = 2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{20}{7^3}+\frac{30}{7^4}+.....$ Then $4S$ is equal to:

(A) $\left(\frac{7}{3}\right)^2$ **(B)** $\frac{7^3}{3^2}$ **(C)** $\left(\frac{7}{3}\right)^3$ **(D)** $\frac{7^2}{3^3}$

6. If $a_1, a_2, a_3 .... and\, b_1, b_2, b_3$ .... are A.P., and $a_1 = 2, a_{10} = 3$, $a_1b_1 = 1 = a_{10}b_{10}$, then a_4b_4 is equal to:

(A) $\frac{35}{27}$ **(B)** 1 **(C)** $\frac{27}{28}$ **(D)** $\frac{28}{27}$

7. **If** $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a|<1, |b|<1, |c|<1$, $abc \neq 0$, then:

(A) x, y, z are in A.P. **(B)** x, y, z are in G.P.

(C) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P. **(D)** $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 1-(a+b+c)$

8. If the sum of the first ten terms of the series $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+........$ is $\frac{m}{n}$, where m and n are co-prime numbers, then $m+n$ is equal to _______.

9. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the least mean is $1:7$ and $a+n = 33$, then the value of n is:

(A) 21 **(B)** 22 **(C)** 23 **(D)** 24

10. Let for $n = 1, 2, ..., 50, S_n$ be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$ is equal to ________.

11. Let $A_1, A_2, A_3, ...$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to:

(A) 33 **(B)** 37 **(C)** 43 **(D)** 47

12. The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + ...$ is equal to:

(A) $\frac{425}{216}$ **(B)** $\frac{429}{216}$ **(C)** $\frac{288}{125}$ **(D)** $\frac{280}{125}$

13. Let 3, 6, 9, 12, ... upto 78 terms and 5, 9, 13, 17... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to _____.

14. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to:

(A) $\frac{6}{343}$ **(B)** $\frac{7}{216}$ **(C)** $\frac{8}{343}$ **(D)** $\frac{49}{216}$

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Archive - JEE Main & Advanced

Complex Numbers	Class - XI | Mathematics

JEE Main 2021

1. If the least and the largest real values of α, for which the equation $z+\alpha|z-1|+2i=0$ $(z \in C \text{ and } i=\sqrt{-1})$ has a solution, are p and q respectively; then $4(p^2+q^2)$ is equal to _______.

2. Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}=k$, and $n=[|k|]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to _________ .

3. Let the lines $(2-i)z=(2+i)\overline{z}$ and $(2+i)z+(i-2)\overline{z}-4i=0$, (here $i^2=-1$) be normal to a circle C. If the line $iz+\overline{z}+1+i=0$ is tangent to this circle C, then its radius is :

(A) $\frac{3}{\sqrt{2}}$ **(B)** $3\sqrt{2}$ **(C)** $\frac{1}{2\sqrt{2}}$ **(D)** $\frac{3}{2\sqrt{2}}$

4. If $\alpha, \beta \in R$ are such that $1-2i$ (here $i^2=-1$) is a root of $z^2+\alpha z+\beta=0$, then $(\alpha-\beta)$ is equal to :

(A) 3 **(B)** –7 **(C)** –3 **(D)** 7

5. The sum of 162[th] power of the roots of the equation $x^3-2x^2+2x-1=0$ is _______.

6. Let z be those complex numbers which satisfy $|z+5|\le 4$ and $z(1+i)+\overline{z}(1-i)\ge -10, i=\sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha+\beta)$ is __________ .

7. Let S_1, S_2 and S_3 be three sets defined as :

$S_1=\{z \in C : |z-1| \le \sqrt{2}\}$

$S_2=\{z \in C : \text{Re}(1-i) \ge 1\}$

$S_3=\{z \in C : \text{Im}(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$.

(A) has exactly three elements **(B)** is a singleton
(C) has exactly two elements **(D)** has infinitely many elements

8. If the equation $a|z|^2+\overline{\alpha}z+\alpha\overline{z}+d=0$ represents a circle where a, d are real constants, then which of the following condition is correct?

(A) $|\alpha|^2-ad>0$ and $a \in R-\{0\}$ **(B)** $|\alpha|^2-ad \ne 0$

(C) $\alpha=0, a, d \in R^+$ **(D)** $|\alpha|^2-ad \ge 0$ and $a \in R$

9. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is ___________ .

10. The least value of $|z|$ where z is complex number which satisfies the inequality $\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}} |5\sqrt{7} + 9i|, i = \sqrt{-1},$ is equal to:

(A) 8 **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3

11. The area of the triangle with vertices $A(z), B(iz) and\ C(z + iz)$ is:

(A) $\frac{1}{2}|z + iz|^2$ **(B)** 1 **(C)** $\frac{1}{2}|z|^2$ **(D)** $\frac{1}{2}$

12. Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to:

(A) $\frac{1}{2}$ **(B)** 4 **(C)** 2 **(D)** $\frac{1}{4}$

13. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + x\, g(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to:

14. Let a complex number $z, |z| \neq 1,$ satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2.$ Then, the largest value of $|z|$ is equal to ________.

(A) 5 **(B)** 8 **(C)** 7 **(D)** 6

15. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2, \left|\frac{z+i}{z-3i}\right| = 1$ and $\text{Re}(w)$ has minimum value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to ________.

16. If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega}\right)$ is:

(Here arg(z) denotes the principal argument of complex number z)

(A) $-\frac{3\pi}{4}$ **(B)** $\frac{3\pi}{4}$ **(C)** $-\frac{\pi}{4}$ **(D)** $\frac{\pi}{4}$

17. If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral $\int_0^{\theta} \sin x\, dx$ is equal to:

(A) -1 **(B)** 2 **(C)** 1 **(D)** 0

18. Let C be the set of all complex numbers. Let $S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\}$, $S_2 = \{z \in C \mid \text{Re}(z) \geq 5\}$ and $S_3 = \{z \in C \mid |z - \bar{z}| \geq 8\}$. Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to :

(A) 0 **(B)** 1 **(C)** 2 **(D)** Infinite

19. Let $\mathbb{C}$ be the set of all complex numbers. Let

$S_1 = \{z \in \mathbb{C} : |z-2| \le 1\}$ and

$S_2 = \{z \in \mathbb{C} : z(1+i) + \bar{z}(1-i) \ge 4\}$

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to :

(A) $\frac{3+2\sqrt{2}}{2}$ **(B)** $\frac{5+2\sqrt{2}}{4}$ **(C)** $\frac{3+2\sqrt{2}}{4}$ **(D)** $\frac{5+2\sqrt{2}}{2}$

20. If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to _____.

21. If $S = \left\{z \in \mathbb{C} : \frac{z-i}{z+2i} \in \mathbb{R}\right\}$, then:

(A) S is a straight line in the complex plane **(B)** S is a circle in the complex plane

(C) S contains only one element **(D)** S contains exactly two elements

22. The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$, is a positive integer, is __________.

23. If $(\sqrt{3}+i)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation:

(A) $x^2 - (\sqrt{3}-1)x - \sqrt{3} = 0$

(B) $x^2 + (\sqrt{3}+1)x + \sqrt{3} = 0$

(C) $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

(D) $x^2 + (\sqrt{3}-1)x - \sqrt{3} = 0$

24. Let $z = \frac{1-i\sqrt{3}}{2}, i = \sqrt{-1}$. Then the value of

$21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \ldots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$ is _______.

25. The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with:

(A) centre at $(0, 0)$ and radius $\sqrt{2}$ **(B)** centre at $(0, 1)$ and radius $\sqrt{2}$

(C) centre at $(0, -1)$ and radius $\sqrt{2}$ **(D)** centre at $(0, 1)$ and radius 2

26. If for the complex numbers z satisfying $|z-2-2i| \le 1$, the maximum value of $|3iz+6|$ is attained at $a+ib$, then $a+b$ is equal to:

27. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $|z - 9\sqrt{2} - 2i|^2$ is equal to _______ .

28. If $a_r = \cos\frac{2r\pi}{9} + i\sin\frac{2r\pi}{9}, r = 1, 2, 3, ..., i = \sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to:

(A) a_9 **(B)** $a_1a_9 - a_3a_7$ **(C)** $a_2a_6 - a_4a_8$ **(D)** a_5

29. Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z - 3| = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to _______.

30. Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to:

(A) 1 **(B)** 2

(C) $\frac{4}{3}$ **(D)** $\frac{3}{2}$

JEE Advanced 2021

1. Let $\theta_1, \theta_2, ...\theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + ... + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}, z_k = z_{k-1}e^{i\theta_k}$ for $k = 2, 3, ..., 10$, where $i = \sqrt{-1}$. Consider the statements P and Q given below:

$$P : |z_2 - z_1| + |z_3 - z_2| + ... + |z_{10} - z_9| + |z_1 - z_{10}| \le 2\pi$$

$$Q : |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + ... + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \le 4\pi$$

Then,

(A) P is TRUE and Q is FALSE **(B)** Q is TRUE and P is FALSE

(C) both P and Q are TRUE **(D)** both P and Q are FALSE

2. For any complex number $w = c + id$, let $\arg(w) \in (-\pi, \pi)$, where $i = \sqrt{-1}$, Let α *and* β be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg\left(\frac{z + \alpha}{z + \beta}\right) = \frac{\pi}{4}$, the ordered pair (x, y) lies on the circle $x^2 + y^2 + 5x - 3y + 4 = 0$

Then which of the following statements is (ae) TRUE?

(A) $\alpha = -1$ **(B)** $\alpha\beta = 4$ **(C)** $\alpha\beta = -4$ **(D)** $\beta = 4$

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Complex Numbers	Class - XI | Mathematics

JEE Main 2022

1. The α, β are the roots of the equation.

$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right) + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$, then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is:

(A) $3x^2 - 20x - 12 = 0$ **(B)** $3x^2 - 10x - 4 = 0$

(C) $3x^2 - 10x + 2 = 0$ **(D)** $3x^2 - 20x + 16 = 0$

2. Let $A = \{z \in C : 1 \le |z - (1+i)| \le 2\}$ and $B = \{z \in A : |z - (1-i)| = 1\}$. Then, B :

(A) is an empty set **(B)** contains exactly two elements

(C) contains exactly three elements **(D)** is an infinite set

3. For $z \in C$ if the minimum value of $\left(\left|z - 3\sqrt{2}\right| + \left|z - p\sqrt{2}i\right|\right)$ is $5\sqrt{2}$, then a value of p is ________.

(A) 3 **(B)** $\frac{7}{2}$ **(C)** 4 **(D)** $\frac{9}{2}$

4. Let $S = \{z \in \mathbb{C} : |z - 3| \le 1$ and $z(4 + 3i) + \bar{z}(4 - 3i) \le 24\}$. If $\alpha + i\beta$ is the point in S which is closest to $4i$, then $25(\alpha + \beta)$ is equal to ______.

5. Let z_1 and z_2 be two complex numbers such that $\bar{z}_1 = i\bar{z}_2$ and $\arg\left(\frac{z_1}{\bar{z}_2}\right) = \pi$. Then

(A) $\arg z_2 = \frac{\pi}{4}$ **(B)** $\arg z_2 = -\frac{3\pi}{4}$ **(C)** $\arg z_1 = \frac{\pi}{4}$ **(D)** $\arg z_1 = -\frac{3\pi}{4}$

6. Let a circle C in complex plane pass through the point $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z (\ne z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then $\arg(z)$ is equal to:

(A) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$ **(B)** $\tan^{-1}\left(\frac{24}{7}\right) - \pi$

(C) $\tan^{-1}(3) - \pi$ **(D)** $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

7. If $z^2+z+1=0,\ z\in C.$ then $\left|\sum_{n=1}^{15}\left(z^n+(-1)^n\frac{1}{z^n}\right)^2\right|$ is equal to _______.

8. Let $A=\left\{z\in C:\left|\frac{z+1}{z-1}\right|<1\right\}$ and $B=\left\{z\in C:\arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}\right\}$.

Then $A\cap B$ is:

(A) A portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(B) A portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(C) An empty set

(D) A portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

9. The number of points of intersection of $\left|z-(4+3i)\right|=2$ *and* $|z|+|z-4|=6,\ z\in\mathbb{C}$, is:

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

10. Let for some real numbers α *and* β $a=\alpha-i\beta$. If the system of equations $4ix+(1+i)y=0$ and $8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)x+\bar{a}y=0$ has more than one solution, then $\frac{\alpha}{\beta}$ is equal to:

(A) $-2+\sqrt{3}$ **(B)** $2-\sqrt{3}$ **(C)** $2+\sqrt{3}$ **(D)** $-2-\sqrt{3}$

11. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z}=iz^2$ is:

(A) $\frac{3\sqrt{3}}{4}$ **(B)** $\frac{3\sqrt{3}}{2}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{3}{4}$

12. Let $A=\begin{pmatrix}1+i & 1\\ -i & 0\end{pmatrix}$ where $i=\sqrt{-1}$.

Then, the number of elements in the set $\left\{n\in\{1,2,..,100\}:A^n=A\right\}$ is __________.

13. Sum of squares of modulus of all the complex numbers z satisfying $\bar{z}=iz^2+z^2-z$ is equal to __________.

14. The number of elements in the set $\{z=a+ib\in C: a,b\in Z \text{ and } 1<|z-3+2i|<4\}$ is _______.

15. Let α be a root of the equation $1+x^2+x^4=0$. Then the value of $\alpha^{1011}+\alpha^{2022}-\alpha^{3033}$ is equal to:

(A) 1 **(B)** α **(C)** $1+\alpha$ **(D)** $1+2\alpha$

16. Let *arg*(*z*) represent the principal argument of the complex number *z*. Then, |*z*| = 3 and *arg*(*z* – 1) – *arg*(*z* + 1) $=\pi/4$ intersect.

(A) Exactly at one point **(B)** Exactly at two points

(C) Nowhere **(D)** At infinitely many points

17. Let α and β be the roots of the equation $x^2+(2i-1)=0$ Then, the value of $|\alpha^8+\beta^8|$ is equal to:

(A) 50 **(B)** 250 **(C)** 1250 **(D)** 1500

18. Let $S=\{z\in C:|z-2|\le 1, z(1+i)+\bar{z}(1-i)\le 2\}$. Let $|z-4i|$ attains minimum and maximum values, respectively, at $z_1\in S$ and $z_2\in S$. If $5\left(|z_1|^2+|z_2|^2\right)=\alpha+\beta\sqrt{5}$. Where α and β are integers, then the value of $\alpha+\beta$ is equal to _________.

19. Let the minimum value v_0 of $v=|z|^2+|z-3|^2+|z-6i|^2, z\in\mathbb{C}$ is attained at $z=z_0$. Then $\left|2z_0^2-\bar{z}_0^3+3\right|^2+v_0^2$ is equal to:

(A) 1000 **(B)** 1024 **(C)** 1105 **(D)** 1196

20. Let $S=\{z\in\mathbb{C}:z^2+\bar{z}=0\}$. Then $\sum\limits_{z\in S}(\text{Re}(z)+\text{Im}(z))$ is equal to ______.

21. For $n\in N$, let $S_n=\left\{z\in C:|z-3+2i|=\frac{n}{4}\right\}$ & $T_n=\left\{z\in C:|z-2+3i|=\frac{1}{n}\right\}$. Then the number of elements in the set $\{n\in N:S_n\cap T_n=\phi\}$ is:

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4

22. Let $A=\begin{pmatrix}2 & -1 & -1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{pmatrix}$ & $B=A-I$. If $\omega=\frac{\sqrt{3}i-1}{2}$, then the number of elements in the set $\{n\in\{1,2,\ldots,100\}:A^n+(\omega B)^n+A+B\}$ is equal to _______.

23. Let $z=a+ib$, $b\ne 0$ be complex numbers satisfying $z^2=\bar{z}\cdot 2^{1-|z|}$. Then the least value of $n\in N$, such that $z^n=(z+1)^n$, is equal to ___________.

24. If $z\ne 0$ be a complex number such that $\left|z-\frac{1}{z}\right|=2$, then the maximum value of $|z|$ is:

(A) $\sqrt{2}$ **(B)** 1 **(C)** $\sqrt{2}-1$ **(D)** $\sqrt{2}+1$

25. Let $S=\{z=x+iy:|z-1+i|\ge|z|,|z|<2,|z+i|=|z-1|\}$. Then the set of all values of x, for which $w=2x+iy\in S$ for some $y\in\mathbb{R}$, is:

(A) $\left(-\sqrt{2},\frac{1}{2\sqrt{2}}\right]$ **(B)** $\left(-\frac{1}{\sqrt{2}},\frac{1}{4}\right]$ **(C)** $\left(-\sqrt{2},\frac{1}{2}\right]$ **(D)** $\left(-\frac{1}{\sqrt{2}},\frac{1}{2\sqrt{2}}\right]$

26. If $z=x+iy$ satisfies $|z|-2=0$ and $|z-i|-|z+5i|=0$, then

(A) $x+2y-4=0$ **(B)** $x^2+y-4=0$

(C) $x+2y+4=0$ **(D)** $x^2-y+3=0$

27. Let S be the set of all $(\alpha,\beta), \pi < \alpha,\beta < 2\pi,$ for which the complex number $\dfrac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary and $\dfrac{1 + i\cos\beta}{1 - 2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta} = \sin 2\alpha + i\cos 2\beta, (\alpha,\beta) \in S.$

Then $\displaystyle\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \frac{1}{i\bar{Z}_{\alpha\beta}}\right)$ is equal to:

(A) 3 **(B)** $3i$ **(C)** 1 **(D)** $2 - i$

28. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $\text{Re}(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true?

(A) $\arg z_2 = \pi - \tan^{-1} 3$ **(B)** $\arg(z_1 - 2z_2) = -\tan^{-1}\dfrac{4}{3}$

(C) $|z_2| = \sqrt{10}$ **(D)** $|2z_1 - z_2| = 5$

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Permutation and Combination	Class - XI | Mathematics

JEE Main 2021

1. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is:

(A) 1625 **(B)** 575 **(C)** 560 **(D)** 1050

2. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is ______ .

3. The total number of positive solutions (x, y, z) such that $xyz = 24$ is :

(A) 30 **(B)** 45 **(C)** 36 **(D)** 24

4. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is _______.

5. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$. Then:

(A) $2y = 91x$ **(B)** $y = 91x$ **(C)** $2y = 273x$ **(D)** $y = 273x$

6. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ________ .

7. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is :

(A) 35 **(B)** 82 **(C)** 77 **(D)** 42

8. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}, y > z$. Then the number of odd divisors of n, including 1, is :

(A) $6x$ **(B)** 6 **(C)** 11 **(D)** 12

9. Let $A = \{1, 2, 3, \ldots, 10\}$ and $f : A \to A$ be defined as $f(k) = \begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$

Then the number of possible functions $g : A \to A$ such that $gof = f$ is:

(A) 10^5 **(B)** $5!$ **(C)** 5^5 **(D)** ${}^{10}C_5$

10. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ___________ .

11. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:

(A) 122234 **(B)** 122664 **(C)** 26664 **(D)** 22264

12. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is__________.

13. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to:

(A) 1890 **(B)** 1173 **(C)** 717 **(D)** 795

14. Team 'A' consists of 7 boys and *n* girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then *n* is equal to :

(A) 5 **(B)** 2 **(C)** 4 **(D)** 6

15. If the series *AB*, *BC* and *CA* of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these as vertices, is equal to :

(A) 364 **(B)** 333 **(C)** 360 **(D)** 240

16. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways , a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is _____.

17. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to ______.

18. Let *A* = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to _____.

19. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students Of class 10 and 11 is 100 k, then k is equal to __________ .

20. If ${}^{n}P_{r} = {}^{n}P_{r+1}$ and ${}^{n}C_{r} = {}^{n}C_{r-1}$, then the value of r is equal to:

(A) 3 **(B)** 4 **(C)** 2 **(D)** 1

21. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to _______.

22. Let *n* a non-negative integer. Then the number of divisors of the form "4*n* + 1" of the number $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$ is equal to __________.

23. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is ________.

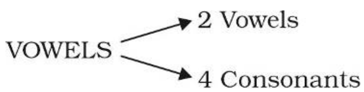
24. The number of three-digit even numbers, formed by the digits 0, 1, 3,4, 6, 7 if the repetition of digits is not allowed, is ________.

25. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is:

26. Let $P_1, P_2, \ldots . P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is:

(A) 419 **(B)** 455 **(C)** 12 **(D)** 443

27. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _________ .



JEE Advanced 2021

1. Let

$$S_1 = \{(i,\ j,\ k) : i,\ j, k \in \{1, 2, \ldots, 10\}\}$$

$$S_2 = \{(i,\ j) : 1 \le i < j + 2 \le 10, i,\ j \in \{1,\ 2, \ldots, 10)\},$$

$$S_3 = \{(i,\ j,\ k,\ l) : 1 \le i < j < k < l,\ i,\ j, k, l \in \{1,\ 2,\ \ldots, 10\}\}$$

and

$S_4 = \{(i,\ j, k, l) : i,\ j, k$ and l are distinct elements in {1, 2, ... 10}}.

If the total number of elements in the set S_r is $n_r, r = 1, 2, 3, 4,$ then which of the following statements is (are) TRUE?

(A) $n_1 = 1000$ **(B)** $n_2 = 44$ **(C)** $n_3 = 220$ **(D)** $\frac{n_4}{12} = 420$

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Permutation and Combination	Class - XI | Mathematics

JEE Main 2022

1. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is ________.

2. The sum of all the elements of the set $\{\alpha \in \{1, 2, ...,100\} : HCF(\alpha, 24) = 1\}$ is _______.

3. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, – 2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is _________.

4. The total number of three-digit numbers, with one digit repeated exactly two times, is __________.

5. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is___________.

6. The total number of 3-digit numbers, whose greatest common divisor with 36 is 2, is _______.

7. There are ten boys $B_1, B_2, ..., B_{10}$ and five girls $G_1, G_2, ..., G_5$ in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is _________.

8. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _____.

9. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to:

(A) 205 **(B)** 615 **(C)** 510 **(D)** 430

10. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is _________.

11. If the system of linear equations.

$$2x + 3y - z = -2$$
$$x + y + z = 4$$
$$x - y + |\lambda| z = 4\lambda - 4$$

Where $\lambda \in R$, has no solution, then:

(A) $\lambda = 7$ **(B)** $\lambda = -7$ **(C)** $\lambda = 8$ **(D)** $\lambda^2 = 1$

12. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is:

(A) 36 **(B)** 48 **(C)** 60 **(D)** 72

13. The total number of four digits numbers such that each of first three digits is divisible by the last digit, is equal to ______.

14. Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, ..., 100\}$ for $1 \le i \le 4$ and $b_i \ne b_j$ for $i \ne j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1b_2b_3b_4$ is equal to ________.

15. Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from $\{A, B, C, D, E\}$ or a number from $\{1, 2, 3, 4, 5\}$ with the repetition of characters allowed. If the number of passwords in S whose at least one character is a number from $\{1, 2, 3, 4, 5\}$ is $\alpha \times 5^6$, then α is equal to _______.

16. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is ______.

17. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is ________.

18. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is _____.

19. The number of 5-digit natural numbers, such that the product of their digits is 36, is _______.

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Binomial Theorem	Class - XI | Mathematics

JEE Main 2021

1. The value of $-{}^{15}C_1 + 2\cdot{}^{15}C_2 - 3\cdot{}^{15}C_3 + \ldots\ldots - 15\cdot{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots\ldots + {}^{14}C_{11}$ is:

(A) $2^{13} - 14$ **(B)** $2^{13} - 13$ **(C)** 2^{14} **(D)** $2^{16} - 1$

2. If $n \geq 2$ is a positive integer, then the sum of the series

${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^nC_2)$ is:

(A) $\dfrac{n(n+1)^2(n+2)}{12}$ **(B)** $\dfrac{n(n+1)(2n+1)}{6}$ **(C)** $\dfrac{n(n-1)(2n+1)}{6}$ **(D)** $\dfrac{n(2n+1)(3n+1)}{6}$

3. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$. The maximum value of k for which the sum

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-1} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to _____.

4. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is ______ .

5. The maximum value of the term independent of 't' in the expansion of $\left(tx^{1/5} + \dfrac{(1-x)^{1/10}}{t}\right)^{10}$ where $x \in (0, 1)$ is :

(A) $\dfrac{2.10!}{3(5!)^2}$ **(B)** $\dfrac{10!}{3(5!)^2}$ **(C)** $\dfrac{10!}{\sqrt{3}(5!)^2}$ **(D)** $\dfrac{2.10!}{\sqrt{3}(5!)^2}$

6. Let $m, n \in N$ and $\gcd(2, n) = 1$. If $30\binom{30}{0} + 29\binom{30}{1} + \ldots\ldots + 2\binom{30}{28} + 1\binom{30}{29} = n \cdot 2^m$, then $n + m$ is equal to _______. $\left(\text{Here} \binom{n}{k} = {}^nC_k\right)$

7. The value of $\sum_{r=0}^{6} ({}^6C_r \cdot {}^6C_{6-r})$ is equal to :

(A) 924 **(B)** 1024 **(C)** 1124 **(D)** 1324

8. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \dfrac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to _______.

9. Let $(1+x+2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$. Then, $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to:

(A) $2^{19}(2^{20}-21)$ **(B)** $2^{19}(2^{20}+21)$

(C) $2^{20}(2^{20}-21)$ **(D)** $2^{20}(2^{20}+21)$

10. Let n be a positive integer. Let $A = \sum_{k=0}^{n} (-1)^k \, {}^nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$. If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to __________ .

11. If $(2021)^{3762}$ is divided by 17, then the remainder is____________.

12. If the fourth term in the expansion of $(x + x^{\log_2 x})^7$ is 4480, then the value of x where $x \in N$ is equal to :

(A) 3. **(B)** 1 **(C)** 2 **(D)** 4

13. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$. If $\sum_{k=0}^{10} (2^2 + 3k)\, {}^nC_k = \alpha.3^{10} + \beta.2^{10}, \alpha, \beta \in R$, then $\alpha + \beta$ is equal to:

14. If $\sum_{r=1}^{10} r!\,(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$, then the value of α is equal to:

15. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to:

16. If n is the number of irrational terms in the expansion of $(3^{1/4} + 5^{1/8})^{60}$, then $(n-1)$ is divisible by :

(A) 7 **(B)** 30 **(C)** 8 **(D)** 26

17. Let $[x]$ denote greatest integer less than or equal to x. If for $n \in N$, $(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$, then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ is equal to :

(A) 1 **(B)** 2^{n-1} **(C)** 2 **(D)** n

18. The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2 + x + 1)^{100}$ is:

(A) ${}^{100}C_{16}$ **(B)** ${}^{100}C_{15}$ **(C)** $-{}^{100}C_{15}$ **(D)** $-{}^{100}C_{16}$

19. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ is _______.

20. For the natural numbers m, n, if $(1-y)^m (1+y)^n = 1 + a_1 y + a_2 y^2 + \ldots + a_{m+n} y^{m+n}$ and $a_1 = a_2 = 10$, then the value of $(m+n)$ is equal to:

(A) 100 **(B)** 80 **(C)** 64 **(D)** 88

21. The number of elements in the set $\{n \in \{1, 2, 3, \ldots\ldots 100\} \mid (11)^n > (10)^n + (9)^n\}$ is ______.

22. If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to ______.

23. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identify $\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \ldots\ldots + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$, then the value of γ is:

(A) $\frac{a+b}{3a^2}$ **(B)** $\frac{a^2+b}{3a^3}$ **(C)** $\frac{a+b^2}{3a^3}$ **(D)** $\frac{b^2}{3a^3}$

24. The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is:

25. The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equal to __________ .

26. The lowest integer which is greater than $\left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$ is ______.

(A) 2 **(B)** 1 **(C)** 3 **(D)** 4

27. If the greatest value of the term independent of 'x' in the expansion of $\left(x \sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to:

(A) –2 **(B)** –1 **(C)** 1 **(D)** 2

28. The sum of all those terms which are rational numbers in the expansion of $\left(2^{1/3} + 3^{1/4}\right)^{12}$ is:

(A) 27 **(B)** 89 **(C)** 35 **(D)** 43

29. Let $n \in N$ and $[x]$ denote the greatest integer less than or equal to x. If the sum of $(n+1)$ terms $^nC_0, 3 \cdot {^nC_1}, 5 \cdot {^nC_2}, 7 \cdot {^nC_3}, \ldots\ldots\ldots$ is equal to $2^{100} \cdot 100$, then $2\left[\frac{n-1}{2}\right]$ is equal to ______.

30. If the co-efficients of x^7 and x^8 in the expansion of $\left(2 + \frac{x}{3}\right)^n$ are equal, then the value of n is equal

to _____.

31. If the coefficient of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to :

(A) -1 **(B)** 1 **(C)** 2 **(D)** -2

32. A possible value of 'x', for which the ninth term in the expansion of

$$\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{\left(-\frac{1}{8}\right)\log_3(5^{x-1}+1)}\right\}^{10}$$

in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3(5^{x-1}+1)}$ is equal to 180, is:

(A) 0 **(B)** 1 **(C)** -1 **(D)** 2

33. $\sum_{k=0}^{20}({}^{20}C_k)^2$ is equal to:

(A) ${}^{40}C_{21}$ **(B)** ${}^{40}C_{20}$ **(C)** ${}^{41}C_{20}$ **(D)** ${}^{40}C_{19}$

34. Let $\binom{n}{k}$ denote ${}^nC_k$ and $\begin{bmatrix} n \\ k \end{bmatrix}=\begin{cases}\binom{n}{k}, & \text{if } 0 \le k \le n \\ 0, & \text{otherwise}\end{cases}$. If $A_k=\sum_{i=0}^{9}\binom{9}{i}\begin{bmatrix} 12 \\ 12-k+i \end{bmatrix}+\sum_{i=0}^{8}\binom{8}{i}\begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$ and $A_4-A_3=190p$, then p is equal to __________.

35. If the sum of the coefficients in the expansion of $(x+y)^n$ is 4096, then the greatest coefficient in the expansion is:

36. If $\left(\frac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$, then k is equal to ______ .

37. $3\times7^{22}+2\times10^{22}-44$ when divided by 18 leaves the remainder _______.

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Archive - JEE Main & Advanced

Binomial Theorem	Class - XI | Mathematics

JEE Main 2022

1. Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$, in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6}:1$. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then α is equal to_________.

2. The remainder when 3^{2022} is divided by 5 is:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

3. The coefficient of x^{101} in the expression.

$(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+......+x^{500}, x>0$, is:

(A) ${}^{501}C_{101}(5)^{399}$ **(B)** ${}^{501}C_{101}(5)^{400}$ **(C)** ${}^{501}C_{100}(5)^{400}$ **(D)** ${}^{500}C_{101}(5)^{399}$

4. If the sum of the co-efficients of all the positive even powers of x in the binomial expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10}-\beta\cdot 3^9$, then β is equal to __________.

5. If $\frac{1}{2\cdot 3^{10}}+\frac{1}{2^2\cdot 3^9}+......+\frac{1}{2^{10}\cdot 3}=\frac{K}{2^{10}\cdot 3^{10}}$, then the remainder when K is divided by 6 is:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5

6. Let C_r denote the binomial coefficient of x^r in the expansion of $(1+x)^{10}$. If for $\alpha, \beta \in R$

$C_1+3\cdot 2C_2+5\cdot 3C_3+......$ upto 10 terms $=\frac{\alpha\times 2^{11}}{2^\beta-1}\left(C_0+\frac{C_1}{2}+\frac{C_2}{3}+.....\text{ upto10 terms}\right)$ then value of $\alpha+\beta$ is equal to _________.

7. For a natural number n, let $\alpha_n=19^n-12^n$. Then, the value of $\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}$ is _______.

8. If $A = \sum_{n=1}^{\infty} \frac{1}{\left(3+(-1)^n\right)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{\left(3+(-1)^n\right)^n}$, then $\frac{A}{B}$ is equal to:

(A) $\frac{11}{9}$ **(B)** 1 **(C)** $-\frac{11}{9}$ **(D)** $-\frac{11}{3}$

9. If $\left({}^{40}C_0\right)+\left({}^{41}C_1\right)+\left({}^{42}C_2\right)+....+\left({}^{60}C_{20}\right)=\frac{m}{n}\,{}^{60}C_{20}$, m and n are coprime, then $m+n$ is equal to_______.

10. The remainder when $(2021)^{2023}$ is divided by 7 is:

(A) 1 **(B)** 2 **(C)** 5 **(D)** 6

11. If the sum of the coefficients of all the position powers of x, in the Binomial expansion of $\left(x^n + \frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible values of n is __________.

12. Let x be a random variable having binomial distribution $B(7, p)$. If $P(X=3)=5P(X=4)$, then the sum of the mean and the variance of X is:

(A) $\frac{105}{16}$ **(B)** $\frac{7}{16}$ **(C)** $\frac{77}{36}$ **(D)** $\frac{49}{16}$

13. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{1/4}}+\frac{\sqrt{5}}{x^{1/3}}\right)^{60}$ is $5^k l$, where $l, k \in N$ and t is co-prime, to 5, then k is equal to_______.

14. The term independent of x in the expansion of $\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}, x \neq 0$ is:

(A) $\frac{7}{40}$ **(B)** $\frac{33}{200}$ **(C)** $\frac{39}{200}$ **(D)** $\frac{11}{50}$

15. If $\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)}$. where $\alpha \in R$, then the value of 16α is equal to:

(A) 1411 **(B)** 1320 **(C)** 1615 **(D)** 1855

16. The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3+\frac{3}{x^k}\right)^{12}, x \neq 0$ *is* $2^8.\ell$ where ℓ is an odd integer is _______.

17. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then $\alpha - \beta$ is equal to:

(A) $1+{}^nC_2(8-5)+{}^nC_3(8^2-5^2)+\ldots+{}^nC_n(8^{n-1}-5^{n-1})$

(B) $1+{}^nC_3(8-5)+{}^nC_4(8^2-5^2)+\ldots+{}^nC_n(8^{n-2}-5^{n-2})$

(C) ${}^nC_3(8-5)+{}^nC_4(8^2-5^2)+\ldots+{}^nC_n(8^{n-2}-5^{n-2})$

(D) ${}^nC_4(8-5)+{}^nC_5(8^2-5^2)+\ldots+{}^nC_n(8^{n-3}-5^{n-3})$

18. Let the coefficients of x^{-1} *and* x^{-3} in the expansion of $\left(2x^{1/5}-\frac{1}{x^{1/5}}\right)^{15}$, $x>0$, be m and n respectively. If r is a positive integer such that $mn^2={}^{15}C_r\cdot 2^r$, then the value of r is equal to ______.

19. If the constant term in the expansion of $\left(3x^3-2x^2+\frac{5}{x^5}\right)^{10}$ is $2^k.l$, where l is an odd integer, then the value of k is equal to:

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9

20. The remainder when $(11)^{1011}+(1011)^{11}$ is divided by 9 is:

(A) 1 **(B)** 4 **(C)** 6 **(D)** 8

21. The remainder when $(2021)^{2022}+(2022)^{2021}$ is divided by 7 is:

(A) 0 **(B)** 1 **(C)** 2 **(D)** 6

22. The remainder when $7^{2022}+3^{2022}$ is divided by 5 is:

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4

23. If the maximum value of the term independent of t in the expansion of $\left(t^2x^{1/5}+\frac{(1-x)^{1/10}}{t}\right)^{15}$, $x\ge 0$, is K, then $8K$ is equal to ______.

24. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4$, $(1-3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6$, $\beta>0$, respectively from the first three terms of an A.P. If d is the common difference of this A.P., then $50-\frac{2d}{\beta^2}$ is equal to __________.

25. If $1+(2+{}^{49}C_1+{}^{49}C_2+....+{}^{49}C_{49})({}^{50}C_2+{}^{50}C_4+....+{}^{50}C_{50})$ is equal to $2^n\cdot m$, where m is odd, then $n+m$ is equal to _____________.

26. $\sum_{r=1}^{20}\left(r^2+1\right)(r!)$ is equal to:

(A) 22! – 21! **(B)** 22! – 2 (21!) **(C)** 21! – 2 (20!) **(D)** 21! – 20!

27. If $\sum_{k=1}^{10}K^2\left(10_{C_K}\right)^2=22000L$, then L is equal to ________.

28. $\sum_{i,\ j=0}^{n} {}^{n}C_i \, {}^{n}C_j$ is equal to $i \neq j$.

(A) $2^{2n} - {}^{2n}C_n$

(B) $2^{2n-1} - {}^{2n-1}C_{n-1}$

(C) $2^{2n} - \frac{1}{2}\,{}^{2n}C_n$

(D) $2^{n-1} + {}^{2n-1}C_n$

29. Let for the 9th term in the binomial expansion of $(3+6x)^n$, in the increasing powers of $6x$, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0. If k is the ratio of the coefficient of x^6 to the coefficient of x^3, then $k + n_0$ is equal to:

30. If the coefficients of x and x^2 in the expansion of $(1+x)^p (1-x)^q$, $p, q \leq 15$, are –3 and –5 respectively, then the coefficient of x^3 is equal to _____.

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Archive - JEE Main & Advanced

Straight Line	Class - XI | Mathematics

JEE Main 2021

1. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C ae placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(A) All the three **(B)** A only **(C)** C only **(D)** B only

2. The image of the point (3, 5) in the line $x - y + 1 = 0,$ lies on :

(A) $(x-4)^2 + (y+2)^2 = 16$ **(B)** $(x-2)^2 + (y-4)^2 = 4$

(C) $(x-2)^2 + (y-2)^2 = 12$ **(D)** $(x-4)^2 + (y-4)^2 = 8$

3. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a :

(A) Equilateral triangle **(B)** Right angled triangle

(C) Isosceles triangle **(D)** None of the above

4. The equation of one of the straight lines which passes through the point (1, 3) and makes an angle $\tan^{-1}(\sqrt{2})$ with the straight line, $y + 1 = 3\sqrt{2}x$ is:

(A) $5\sqrt{2}x + 4y - (15 + 4\sqrt{2}) = 0$ **(B)** $4\sqrt{2}x - 5y - (5 + 4\sqrt{2}) = 0$

(C) $4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$ **(D)** $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$

5. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is:

(A) 3 **(B)** 1 **(C)** 0 **(D)** 2

6. Let $A(-1, 1), B(3, 4)$ and $C(2, 0)$ be given three points. A line $y = mx, m > 0,$ intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the area of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to:

(A) 1 **(B)** 2 **(C)** 3 **(D)** $\frac{4}{15}$

7. In a triangle PQR, the co-ordinates of the points P and Q are (-2,4)and (4, –2) respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of ΔPQR is:

(A) (0, 2) **(B)** (–2, –2) **(C)** (1, 4) **(D)** (–1, 0)

8. Let the centroid of an equilateral triangle *ABC* be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x+y=3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R+r)$ is equal to:

(A) $7\sqrt{2}$ **(B)** $2\sqrt{2}$ **(C)** $\frac{9}{\sqrt{2}}$ **(D)** $3\sqrt{2}$

9. Let $\tan\alpha, \tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}, n \in N$ be the slopes of three line segments *OA*, *OB* and *OC*, respectively, where *O* is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on *y*-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}\right)^2$ is equal to _______.

10. Consider a triangle having vertices $A(-2, 3)$, $B(1, 9)$ and $C(3, 8)$. If a line *L* passing through the circum-center of triangle *ABC*, bisects line *BC*, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is ______.

11. Let the equation of the pair of lines, $y = px$ and $y = qx$, can be written as $(y-px)(y-qx)=0$. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is:

(A) $x^2 + 3xy - y^2 = 0$ **(B)** $x^2 - 3xy - y^2 = 0$

(C) $x^2 + 4xy - y^2 = 0$ **(D)** $x^2 - 3xy + y^2 = 0$

12. The point P (a, b) undergoes the following three transformations successively :

(a) Reflection about the line *y*=*x*

(b) Translation through 2 units along the positive direction of x-axis

(c) Rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of 2a + b is equal to:

(A) 5 **(B)** 9 **(C)** 7 **(D)** 1

13. Two sides of a parallelogram are along the line $4x+5y=0$ and $7x+2y=0$. If the equation of one of the diagonals of the parallelogram is $11x+7y=9$, then other diagonal passes through the point :

(A) (1, 2) **(B)** (2, 2) **(C)** (2, 1) **(D)** (1, 3)

14. Let *A* be a fixed point (0, 6) and *B* be a moving point $(2t, 0)$. Let *M* be the mid-point of *AB* and the perpendicular bisector of *AB* meets the *y*-axis at *C*. The locus of the mid-point *P* of *MC* is:

(A) $2x^2 + 3y - 9 = 0$ **(B)** $3x^2 - 2y - 6 = 0$

(C) $3x^2 + 2y - 6 = 0$ **(D)** $2x^2 - 3y + 9 = 0$

15. Let ABC be a triangle with $A(-3,1)$ *and* $\angle ACB = \theta, 0 < \theta < \frac{\pi}{2}$ If the equation of the median through B is $2x + y - 3 = 0$ and the equation of angle bisector of C is $7x - 4y - 1 = 0$, then $\tan\theta$ is equal to:

(A) $\frac{3}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{4}{3}$ **(D)** 2

16. Let the points of intersections of the lines $x - y + 1 = 0, x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is:

17. A man starts walking from the point $P(-3, 4)$, touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50((PR)^2 + (RQ)^2)$ is equal to:

18. If p and q are the lengths of the perpendiculars from the origin on the lines,

$x\,\text{cosec}\alpha - y\sec\alpha = k\cot 2\alpha$ and $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$ respectively, then k^2 is equal to:

(A) $p^2 + 2q^2$ **(B)** $2p^2 + q^2$ **(C)** $4p^2 + q^2$ **(D)** $p^2 + 4q^2$

19. Let $A(a, 0), B(b, 2b+1)$ and $C(0, b), b \neq 0, |b| \neq 1,$ be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is :

(A) $\frac{-2b}{b+1}$ **(B)** $\frac{2b^2}{b+1}$ **(C)** $\frac{-2b^2}{b+1}$ **(D)** $\frac{2b}{b+1}$

JEE Advanced 2021

1. Consider a triangle Δ whose two sides lie on the x-axis and the line $x + y + 1 = 0$. If the orthocenter of Δ is (1, 1), then the equation of the circle passing through the vertices of the triangle Δ is:

(A) $x^2 + y^2 - 3x + y = 0$ **(B)** $x^2 + y^2 + x + 3y = 0$

(C) $x^2 + y^2 + 2y - 1 = 0$ **(D)** $x^2 + y^2 + x + y = 0$

Question Stem for Question Nos. 2 and 3

Question Stem

Consider the lines L_1 *and* L_2 defined by

$$L_1 : x\sqrt{2} + y - 1 = 0 \text{ and } L_2 : x\sqrt{2} - y + 1 = 0$$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' *and* S'. Let D be the square of the distance between R' *and* S'

2. The value of λ^2 is _____.

3. The value of D is ______.

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Straight Line	Class - XI | Mathematics

JEE Main 2022

1. Let the area of the triangle with vertices $A(1, \alpha), B(\alpha, 0)$ and $C(0, \alpha)$ be 4 sq. units. If the points $(\alpha, -\alpha), (-\alpha, \alpha)$ and (α^2, β) are collinear, then β is equal to:

(A) 64 **(B)** –8 **(C)** –64 **(D)** 512

2. Let l_1 be the line in xy – plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and l_2 be the line in zx – plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to ___________.

3. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid of ΔABC, then $15(\alpha + \beta)$ is equal to:

(A) 39 **(B)** 41 **(C)** 51 **(D)** 63

4. Let a triangle be bounded by the liens $L_1 : 2x + 5y = 10; L_2 : -4x + 3y = 12$ and the line L_3, which passes through the point $P(2, 3)$, intersects L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio $1:3$, then the area of the triangle is equal to:

(A) $\frac{110}{13}$ **(B)** $\frac{132}{13}$ **(C)** $\frac{142}{13}$ **(D)** $\frac{151}{13}$

5. A ray of light passing through the point $P(2, 3)$ reflects on the x-axis at point A and the reflected ray passes through the point $Q(5, 4)$. Let R be the point that divides the line segment AQ internally into the ratio 2:1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to _________.

6. The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is:

(A) $\sqrt{2}$ **(B)** 2 **(C)** $2\sqrt{2}$ **(D)** 4

7. The distance between the two points A and A' which lie on $y = 2$ such that both the line segments AB and $A'B$ (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to:

(A) 10 **(B)** $\frac{48}{5}$ **(C)** $\frac{52}{5}$ **(D)** 3

8. Let the point $P(\alpha, \beta)$ be at a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$, and $L_2 : 8x + 6y + 11 = 0$. If P lies below L_1 and above L_2, then $100(\alpha + \beta)$ is equal to:

(A) –14 **(B)** 42 **(C)** –22 **(D)** 14

9. A line, with the slope greater than one, passes through the point $A(4,3)$ and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line:

(A) $2x + y = 9$ **(B)** $3x - 2y = 7$ **(C)** $x + 2y = 6$ **(D)** $2x - 3y = 3$

10. Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2 + 11a + 3\left(m_1^2 + m_2^2\right) = 220$. If one vertex of the square is $\left(10(\cos\alpha - \sin\alpha), 10(\sin\alpha + \cos\alpha)\right)$, where $\alpha \in \left(0, \frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha - \sin\alpha)x + (\sin\alpha + \cos\alpha)y = 10$, then $72\left(\sin^4\alpha + \cos^4\alpha\right) + a^2 - 3a + 13$ is equal to:

(A) 119 **(B)** 128 **(C)** 145 **(D)** 155

11. Let $A(\alpha, -2), B(\alpha, 6)$ and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a ΔABC. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is NOT correct about ΔABC?

(A) area is 24 **(B)** perimeter is 25

(C) circumradius is 5 **(D)** inradius is 2

12. Let the circumcentre of a triangle with vertices $A(a,3), B(b,5)$ and $C(a,b), ab > 0$ be $P(1,1)$. If the line AP intersects the line BC at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to:

(A) 2 **(B)** $\frac{4}{7}$ **(C)** $\frac{2}{7}$ **(D)** 4

13. The equations of the sides *AB, BC* and *CA* of a triangle *ABC* are $2x + y = 0, x + py = 39$ and $x - y = 3$ respectively and $P(2,3)$ is its circumcentre. Then which of the following is NOT true?

(A) $(AC)^2 = 9p$ **(B)** $(AC)^2 + p^2 = 136$

(C) $32 < area(\Delta ABC) < 36$ **(D)** $34 < area(\Delta ABC) < 38$

14. A point P moves so that the sum of squares of its distances from the points (1, 2) and (–2, 1) is 14. Let $f(x, y) = 0$ be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral $ACBD$ is equal to:

(A) $\frac{9}{2}$ **(B)** $\frac{3\sqrt{17}}{2}$ **(C)** $\frac{3\sqrt{17}}{4}$ **(D)** 9

15. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its orthocentre is $(2, a), -\frac{1}{2} < a < 2$, then p is equal to _______.

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Archive - JEE Main & Advanced

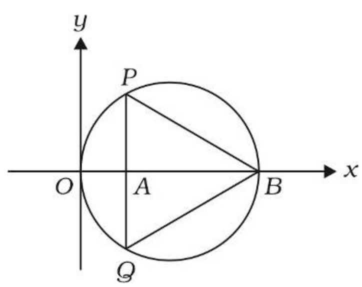
Circle	Class - XI | Mathematics

JEE Main 2021

1. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is chord of another circle 'C', whose centre is at (2, 1), then its radius is _______.

2. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point $(-5, 0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to _________ .

3. In the circle below, let $OA = 1unit, OB = 13unit$ and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is :



(A) $24\sqrt{3}$ **(B)** $26\sqrt{3}$ **(C)** $26\sqrt{2}$ **(D)** $24\sqrt{2}$

4. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to:

(A) 1 **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$

5. Let the normal at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, –3) and $(4, -2\sqrt{2})$, and given that $a - 2\sqrt{2}b = 3$, then $(a^2 + b^2 + ab)$ is equal to _______ .

6. If the area of the triangle formed by the positive x-axis, the normal and the tangent to then circle $(x-2)^2 + (y-3)^2 = 25$ at the point $(5, 7)$ is A, then $24A$ is equal to _________ .

7. For the four circles M, N, O and P, following four equations are given:

Circle M: $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a:

(A) Square **(B)** Rectangle **(C)** Rhombus **(D)** Parallelogram

8. Choose the correct statement about two circles whose equations are given below:

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

(A) circles have only one meeting point **(B)** circles have two meeting points
(C) circles have same centre **(D)** circles have no meeting point

9. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to:

(A) $\sqrt{6}$ **(B)** $\sqrt{7}$ **(C)** $\sqrt{10}$ **(D)** $\sqrt{11}$

10. The minimum distance between any two points P_1 *and* P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 24x - 10y + 160 = 0$ is______________.

11. The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. Then, the radius of the circle is:

(A) $5\sqrt{4}$ **(B)** $3\sqrt{5}$ **(C)** $5\sqrt{3}$ **(D)** $4\sqrt{5}$

12. Choose the incorrect statement about the two circles whose equations are given below:

$x^2 + y^2 - 10x - 10y + 41 = 0$ and $x^2 + y^2 - 16x - 10y + 80 = 0$

(A) Distance between two centres is the average of radii of both the circles.
(B) Circles have two intersection points
(C) Both circle' centres lie inside region of one another
(D) Both circles pass through the centre of each other

13. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points.

(A) $\left(0, \pm\sqrt{3}\right)$ **(B)** $(1, \pm 2)$ **(C)** $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$ **(D)** $\left(2, \pm\frac{3}{2}\right)$

14. Let $ABCD$ be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\,\beta$, where α, β are integers, then $\alpha + \beta$ is equal to _______.

15. Let the tangent to the circle $x^2 + y^2 = 25$ at the point $R(3, 4)$ meet x-axis and y-axis at points P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to :

(A) $\frac{529}{64}$ **(B)** $\frac{625}{72}$ **(C)** $\frac{125}{72}$ **(D)** $\frac{585}{66}$

16. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangent is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denotes by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is :

(A) 11 : 4 **(B)** 2 : 1 **(C)** 3 : 1 **(D)** 9 : 4

17. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point (–4, 1) and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then $a + b$ is equal to:

(A) 11 **(B)** 5 **(C)** 3 **(D)** 7

18. Let the circle $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then:

(A) $100 < C < 165$ **(B)** $\frac{25}{9} < C < \frac{13}{3}$ **(C)** $100 < C < 156$ **(D)** $81 < C < 156$

19. The equation of a circle is $\text{Re}(z^2) + 2(\text{Im}(z))^2 + 2\,\text{Re}(z) = 0$, where $z = x + iy$. A line which passes through the centre of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y intercept equal to ______.

20. Let $A = \{(x, y) \in R \times R \mid 2x^2 + 2y^2 - 2x - 2y = 1\}$, $B = \{(x, y) \in R \times R \mid 4x^2 + 4y^2 - 16y + 7 = 0\}$ and $C = \{(x, y) \in R \times R \mid x^2 + y^2 - 4x - 2y + 5 \le r^2\}$.

Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to :

(A) $\frac{3 + 2\sqrt{5}}{2}$ **(B)** $1 + \sqrt{5}$ **(C)** $\frac{3 + \sqrt{10}}{2}$ **(D)** $\frac{2 + \sqrt{10}}{2}$

21. Two tangents are drawn from the point $P(-1, 1)$ to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of triangle ABD is equal to :

(A) 2 **(B)** 4 **(C)** $(3\sqrt{2} + 2)$ **(D)** $3(\sqrt{2} - 1)$

22. Let P and Q be two distinct points on a circle which has centre at $C(2, 3)$ and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set $\{P, Q\}$ is equal to :

(A) $\{(-1, 5), (5, 1)\}$
(B) $\{(2+2\sqrt{2}, 3+\sqrt{5}), (2-2\sqrt{2}, 3-\sqrt{5})\}$
(C) $\{(4, 0), (0, 6)\}$
(D) $\{(2+2\sqrt{2}, 3-\sqrt{5}), (2-2\sqrt{2}, 3+\sqrt{5})\}$

23. Consider a circle C which touches the y-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the x-axis. Then the radius of the circle C is equal to :

(A) 8 **(B)** $\sqrt{82}$ **(C)** $\sqrt{53}$ **(D)** 9

24. Let the equation $x^2 + y^2 + px + (1-p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer$\}$ is __________ .

25. A circle C touches the line $x = 2y$ at the point (2, 1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is:

(A) 15 **(B)** $7\sqrt{5}$ **(C)** $4\sqrt{15}$ **(D)** $\sqrt{285}$

26. The locus of a point, which moves such that the sum of squares of its distances from the points $(0, 0), (1, 0), (0, 1) (1, 1)$ is 18 unis, is a circle of diameter d. Then d^2 is equal to _______.

27. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$, then the length of this chord is:

(A) 5 **(B)** 7 **(C)** $3\sqrt{5}$ **(D)** $5\sqrt{3}$

28. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is_________ .

29. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangents is $4x + 3y = 10$, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha+\beta)(\gamma+\delta)|$ is equal to _______.

30. Let Z be the set of all integers, $A = \{(x, y) \in Z \times Z : (x-2)^2 + y^2 \leq 4\}$,

$B = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$ and $C = \{(x, y) \in Z \times Z : (x-2)^2 + (y-2)^2 \leq 4\}$. If the total number of relations from $A \cap B$ to $A \cap C$ is 2^p, then the value of p is :

(A) 16 **(B)** 25 **(C)** 49 **(D)** 9

JEE Advanced 2021

Paragraph

Let $M = \{(x, y) \in \mathbb{R} \times \mathbb{R}; x^2 + y^2 \le r^2\}$,

where $r > 0$. Consider the geometric progression $a_n = \frac{1}{2^{n-1}}, n = 1, 2, 3, \ldots\ldots$. Let $S_0 = 0$ and , for $n \ge 1$, let S_n denote the sum of the first n terms of this progression. For $n \ge 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n, and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n.

1. Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then

(A) $k + 2l = 22$ **(B)** $2k + l = 26$ **(C)** $2k + 3l = 34$ **(D)** $3k + 2l = 40$

2. Consider M with $r = \frac{(2^{100} - 1)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is :

(A) 198 **(B)** 199 **(C)** 200 **(D)** 201

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Archive - JEE Main & Advanced

Circle	Class - XI | Mathematics

JEE Main 2022

1. Let a circle $C:(x-h)^2+(y-k)^2=r^2, k>0$, touch the x-axis at (1, 0). If the line $x+y=0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h+k+r$ is equal to ___.

2. A circle touches both the y-axis and the line $x+y=0$. Then the locus of its center is:

(A) $y=\sqrt{2}x$ **(B)** $x=\sqrt{2}y$ **(C)** $y^2-x^2=2xy$ **(D)** $x^2-y^2=2xy$

3. Let a circle C touch the lines $L_1:4x-3y+K_1=0$ and $L_2:4x-3y+K_2=0$, $K_1,K_2\in R$. If a line passing through the centre of the circle C intersects L_1 at $(-1,2)$ and L_2 at $(3,-6)$, then the equation of the circle C is:

(A) $(x-1)^2+(y-2)^2=4$ **(B)** $(x+1)^2+(y-2)^2=4$

(C) $(x-1)^2+(y+2)^2=16$ **(D)** $(x-1)^2+(y-2)^2=16$

4. Let the abscissae of the two points P and Q be the roots of $2x^2-rx+p=0$ and the ordinates of P and Q be the roots of $x^2-sx-q=0$. If the equation of the circle described on PQ as diameter is $2(x^2+y^2)-11x-14y-22=0$, then $2r+s-2q+p$ is equal to ________.

5. Let C be a circle passing through the points $A(2, -1)$ and $B(3, 4)$. The line segment AB is not a diameter of C. If r is the radius of C and its centre lies on the circle $(x-5)^2+(y-1)^2=\frac{13}{2}$, then r^2 is equal to:

(A) 32 **(B)** $\frac{65}{2}$ **(C)** $\frac{61}{2}$ **(D)** 30

6. The set of values of k, for which the circle C: $4x^2+4y^2-12x+8y+k=0$ lies inside the fourth quadrant and the point $\left(1,-\frac{1}{3}\right)$ lies on or inside the circle C, is:

(A) an empty set **(B)** $\left[6,\frac{65}{9}\right]$ **(C)** $\left[\frac{80}{9},10\right]$ **(D)** $\left[9,\frac{92}{9}\right]$

7. Let a circle C of radius 5 lie below the x-axis. The line $L_1 : 4x + 3y + 2 = 0$ passes through the centre P of the circle C and intersects line $L_2 : 3x - 4y - 11 = 0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x - 12y + 51 = 0$ is _______.

8. A rectangle R with end points of one of its sides as $(1,2)$ and $(3,6)$ is inscribed in a circle. If the equation of diameter of the circle is $2x - y + 4 = 0$, then the area of R is ________.

9. If one the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $\left(x - 2\sqrt{2}\right)^2 + \left(y - 2\sqrt{2}\right)^2 = r^2$, then the value of r^2 is equal to __________.

10. If the tangents drawn at the points $O(0, 0)$ and $P\left(1+\sqrt{5}, 2\right)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to:

(A) $\frac{3+\sqrt{5}}{2}$ **(B)** $\frac{4+2\sqrt{5}}{2}$ **(C)** $\frac{5+3\sqrt{5}}{2}$ **(D)** $\frac{7+3\sqrt{5}}{2}$

11. Let a triangle ABC be inscribed in the circle $x^2 - \sqrt{2}(x+y) + y^2 = 0$ such that $\angle BAC = \frac{\pi}{3}$. If the length of side AB is $\sqrt{2}$, then the area of the ΔABC is equal to:

(A) $(\sqrt{2}+\sqrt{6})/3$ **(B)** $(\sqrt{6}+\sqrt{3})/2$ **(C)** $(3+\sqrt{3})/4$ **(D)** $(\sqrt{6}+2\sqrt{3})/4$

12. Let the tangent to the circle $C_1 : x^2 + y^2 = 2$ at the point $M(-1,1)$ intersect the circle $C_2 : (x-3)^2 + (y-2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to:

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{5}{3}$

13. If the circles $x^2 + y^2 + 6x + 8y + 16 = 0$ and $x^2 + y^2 + 2\left(3-\sqrt{3}\right)x + 2\left(4-\sqrt{6}\right)y = k + 6\sqrt{3} + 8\sqrt{6}, k > 0,$ touch internally at the point $P(\alpha, \beta)$, then $\left(\alpha+\sqrt{3}\right)^2 + \left(\beta+\sqrt{6}\right)^2$ is equal to ____________.

14. If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0, g, c \in \mathbb{R}$ passes through the point (6, 1) and its centre lies on the line $x - 2cy = 8$, then the length of intercept made by the circle on x-axis is:

(A) $\sqrt{11}$ **(B)** 4 **(C)** 3 **(D)** $2\sqrt{23}$

15. For $t \in (0, 2\pi)$, if ABC is an equilateral triangle with vertices $A(\sin t, -\cos t), B(\cos t, \sin t)$ and $C(a, b)$ such that its orthocenter lies on a circle with centre $\left(1, \frac{1}{3}\right)$, then $\left(a^2 - b^2\right)$ is equal to :

(A) $\frac{8}{3}$ **(B)** 8 **(C)** $\frac{77}{9}$ **(D)** $\frac{80}{9}$

16. Let C be the centre of the circle $x^2+y^2-x+2y=\frac{11}{4}$ and P be a point on the circle. A line passes through the point C, makes an angle of $\frac{\pi}{4}$ with the line CP and intersects the circle at the points Q and R. Then the area of the triangle $PQR\left(\text{in unit}^2\right)$ is:

(A) 2 **(B)** $2\sqrt{2}$ **(C)** $8\sin\left(\frac{\pi}{8}\right)$ **(D)** $8\cos\left(\frac{\pi}{8}\right)$

17. Let the locus of the centre $(\alpha,\beta), \beta>0,$ of the circle which touches the circle $x^2+(y-1)^2=1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is:

(A) $\frac{32\sqrt{2}}{3}$ **(B)** $\frac{40\sqrt{2}}{3}$ **(C)** $\frac{64}{3}$ **(D)** $\frac{32}{3}$

18. Let the tangents at two points A and B on the circle $x^2+y^2-4x+3=0$ meet at origin $O(0, 0)$. Then the area of the triangle OAB is:

(A) $\frac{3\sqrt{3}}{2}$ **(B)** $\frac{3\sqrt{3}}{4}$ **(C)** $\frac{3}{2\sqrt{3}}$ **(D)** $\frac{3}{4\sqrt{3}}$

19. Let AB be a chord of length 12 of the circle $(x-2)^2+(y+1)^2=\frac{169}{4}$. If tangents drawn to the circle at point A and B intersect at the point P, then five times the distance of point P from chord AB is equal to __________.

20. Let $S=\left\{(x,y)\in N\times N : 9(x-3)^2+16(y-4)^2\le 144\right\}$ and

$T=\left\{(x,y)\in \mathbb{R}\times\mathbb{R} : (x-7)^2+(y-4)^2\le 36\right\}$. Then $n(S\cap T)$ is equal to ___________.

21. Let the abscissae of the two point P and Q on a circle be the roots of $x^2-4x-6=0$ and the ordinates of P and Q be the roots of $y^2+2y-7=0$, If PQ is a diameter of the circle $x^2+y^2+2ax+2by+c=0$, then the value of $(a+b-c)$ is _____.

(A) 12 **(B)** 13 **(C)** 14 **(D)** 16

22. A circle C_1 passes through the origin O and has diameter 4 on the positive x-axis. The line $y=2x$ gives a chord OA of circle C_1. Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point A meets the x-axis at P and y-axis at Q, then $QA : AP$ is equal to:

(A) 1 : 4 **(B)** 1 : 5 **(C)** 2 : 5 **(D)** 1 : 3

23. Let the mirror image of a circle $c_1 : x^2+y^2-2x-6y+\alpha=0$ in line $y=x+1$ be $c_2 : 5x^2+5y^2+10gx+10fy+38=0$. If r is the radius of circle c_2, then $\alpha+6r^2$ is equal to_________.

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Archive - JEE Main & Advanced

Conic Sections	Class - XI | Mathematics

JEE Main 2021

1. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is:

(A) $x = -\frac{a}{2}$ **(B)** $x = 0$ **(C)** $x = \frac{a}{2}$ **(D)** $x = a$

2. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(A) $y^2 = \frac{1}{6\sqrt{3}}x$ **(B)** $x^2 + 9y^2 = 9$ **(C)** $2x^2 - 18y^2 = 9$ **(D)** $x^2 + y^2 = 7$

3. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

(A) (3, 13) **(B)** (1, 5) **(C)** (–2, 8) **(D)** (2, 8)

4. If the curves, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ intersect each other at an angle of 90°, then which of the following relations is TRUE ?

(A) $ab = \frac{c+d}{a+b}$ **(B)** $a - b = c - d$ **(C)** $a + b = c + d$ **(D)** $a - c = b + d$

5. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

(A) (5, 4) **(B)** (−6, 0) **(C)** (4, 5) **(D)** (0, 3)

6. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4(\sqrt{3})k = 0$ is a conic, whose eccentricity is _______.

7. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is:

(A) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ **(B)** $\frac{x^2}{9} - \frac{y^2}{4} = 1$ **(C)** $x^2 - y^2 = 9$ **(D)** $\frac{x^2}{9} - \frac{y^2}{25} = 1$

8. If the curve $x^2+2y^2=2$ intersects the line $x+y=1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is:

(A) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$ **(B)** $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$ **(C)** $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{3}\right)$ **(D)** $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{3}\right)$

9. A line is a common tangent to the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a+c)$ is equal to ________ .

10. Let $A(1, 4)$ and $B(1, -5)$ be two points. Let P be a point on the circle $(x-1)^2+(y-1)^2=1$ such that $(PA)^2+(PB)^2$ have maximum value, then the points, P, A and B lie on :

(A) a straight line **(B)** an ellipse

(C) a parabola **(D)** a hyperbola

11. Let L be a common tangent line to the curves $4x^2+9y^2=36$ and $(2x)^2+(2y)^2=31$. Then the square of the slope of the line L is __________ .

12. A square ABCD has all its vertices on the curve $x^2y^2=1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is ______________ .

13. Let C be the locus of the mirror image of a point on the parabola $y^2=4x$ with respect to the line $y=x$. Then the equation of tangent to C at $P(2, 1)$ is :

(A) $2x+y=5$ **(B)** $x+3y=5$ **(C)** $x-y=1$ **(D)** $x+2y=4$

14. If the points of intersection of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the circle $x^2+y^2=4b$, $b>4$ lie on the curve $y^2=3x^2$, then b is equal to:

(A) 10 **(B)** 5 **(C)** 6 **(D)** 12

15. Let a tangent be drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ where $\theta\in\left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to:

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{8}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{\pi}{6}$

16. Consider a hyperbola $H: x^2-2y^2=4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1), x_1>0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to:

(A) $4\sqrt{6}$ **(B)** $\sqrt{6}-1$ **(C)** $4\sqrt{6}-1$ **(D)** $\frac{7}{\sqrt{6}}-2$

17. If the three normals drawn to the parabola, $y^2=2x$ pass through the point $(a, 0)$ $a\neq 0$, then 'a' must be greater than :

(A) $-\frac{1}{2}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** -1

18. Let L be a tangent to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$, then the value of b is equal to :

(A) 16 **(B)** 11 **(C)** 14 **(D)** 20

19. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

(A) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$ **(B)** $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

(C) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$ **(D)** $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

20. Let the tangent to the parabola S: $y^2 = 2x$ at the point $P(2,2)$ meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to:

(A) 25 **(B)** $\frac{15}{2}$ **(C)** $\frac{35}{2}$ **(D)** $\frac{25}{2}$

21. Let T be the tangent to the ellipse E: $x^2 + 4y^2 = 5$ at the point $P(1,1)$. If the area of the region bounded by the tangent T, ellipse E, lines $x = 1$ and $x = \sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to_____.

22. Let $y = mx + c, m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x+10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m+c)$ is equal to _______.

23. Let P be a variable point on the parabola $y = 4x^2 + 1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line $y = x$ is:

(A) $(3x - y)^2 + 2(x - 3y) + 2 = 0$ **(B)** $(3x - y)^2 + (x - 3y) + 2 = 0$

(C) $2(x - 3y)^2 + (3x - y) + 2 = 0$ **(D)** $2(3x - y)^2 + (x - 3y) + 2 = 0$

24. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha + \beta)$ is equal to _____ .

25. Let a line $L : 2x + y = k, k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to:

(A) 24 **(B)** –24 **(C)** 12 **(D)** –12

26. Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a > b$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is:

(A) $\frac{-1+\sqrt{3}}{2}$ **(B)** $\frac{-1+\sqrt{5}}{2}$ **(C)** $\frac{-1+\sqrt{8}}{2}$ **(D)** $\frac{-1+\sqrt{6}}{2}$

27. Let an ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus $F(\alpha, 0), \alpha > 0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to:

(A) $\frac{4}{3}$ **(B)** $\frac{8}{3}$ **(C)** 3 **(D)** $\frac{16}{3}$

28. Let a parabola P be such that its vertex and focus lie on the positive x-axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) Of ΔSOR is equal to:

(A) $8\sqrt{2}$ **(B)** 32 **(C)** 16 **(D)** $16\sqrt{2}$

29. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is:

(A) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$ **(B)** $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

(C) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$ **(D)** $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

30. If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point:

(A) $(-1, 1)$ **(B)** $(1, 1)$ **(C)** $(\sqrt{2}, 0)$ **(D)** $(\sqrt{3}, 0)$

31. A ray of light through (2, 1) is reflected to a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be :

(A) $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$ **(B)** $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$

(C) $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$ **(D)** $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$

32. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its centre at (3, –4), one focus at (4, –4) and one vertex at (5, –4). If $mx - y = 4$, $m > 0$ is a tangent to the ellipse E, then the value of $5m^2$ is equal to ___________.

33. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab, then k is equal to _________.

34. A tangent and a normal are drawn at the point $P(2, -4)$ on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If $Q(a, b)$ is a point such that $AQBP$ is square, then $2a+b$ is equal to:

(A) –18 **(B)** –20 **(C)** –12 **(D)** –16

35. If $x^2+9y^2-4x+3=0,\ x, y \in \mathbb{R},$ then x and y respectively lie in the intervals:

(A) $[1, 3]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$ **(B)** $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

(C) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $[1, 3]$ **(D)** $[1, 3]$ and $[1, 3]$

36. The point $P(-2\sqrt{6}, \sqrt{3})$ lies on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the points Q and R respectively, then QR is equal to:

(A) 6 **(B)** $4\sqrt{3}$ **(C)** $3\sqrt{6}$ **(D)** $6\sqrt{3}$

37. The locus of the mid points of the chords of the hyperbola $x^2-y^2=4$, which touch the parabola $y^2=8x$, is :

(A) $x^3(x-2)=y^2$ **(B)** $y^3(x-2)=x^2$ **(C)** $y^2(x-2)=x^3$ **(D)** $x^2(x-2)=y^3$

38. On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x+2y=0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5-e^2)$. A is:

(A) 12 **(B)** 6 **(C)** 24 **(D)** 14

39. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y=\frac{1}{2}$. Let P be the point where the parabola meets the line $x=-\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again in the point Q, then $(PQ)^2$ is equal to:

(A) $\frac{25}{2}$ **(B)** $\frac{15}{2}$ **(C)** $\frac{125}{16}$ **(D)** $\frac{75}{8}$

40. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1$ and the circle $x^2+y^2=3$ at their point of intersection in the first quadrant. Then $\tan\theta$ is equal to:

(A) $\frac{2}{\sqrt{3}}$ **(B)** 2 **(C)** $\frac{5}{2\sqrt{3}}$ **(D)** $\frac{4}{\sqrt{3}}$

41. The line $12x\cos\theta+5y\sin\theta=60$ is tangent to which of the following curves?

(A) $25x^2 + 12y^2 = 3600$ **(B)** $x^2 + y^2 = 169$

(C) $144x^2 + 25y^2 = 3600$ **(D)** $x^2 + y^2 = 60$

42. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x – axis at a distance R and S (> R) respectively from the origin, is:

(A) $4(S-R)$ **(B)** $2(S+R)$ **(C)** $4(S+R)$ **(D)** $2(S-R)$

43. Let $A(\sec\theta, 2\tan\theta)$ and $B(\sec\phi, 2\tan\phi)$, where $\theta+\phi=\pi/2$, be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to _______.

44. If two tangents drawn from a point P to the parabola $y^2 = 16(x-3)$ are at right angles, then the locus of point P is :

(A) $x+1=0$ **(B)** $x+3=0$ **(C)** $x+4=0$ **(D)** $x+2=0$

JEE Advanced 2021

1. Let E denote the parabola $y^2 = 8x$. Let $P = (-2, 4)$, and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is (are) **TRUE**?

(A) The triangle PFQ is a right-angled triangle

(B) The triangle QPQ' is a right-angled triangle

(C) The distance between P and F is $5\sqrt{2}$

(D) F lies on the line joining Q and Q'

Question Stem for Question Nos. 2 and 3

Question Stem

Consider the region $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0 \text{ and } y^2 \leq 4 - x\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F. Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

2. The radius of the circle C is ______ .

3. The value of α is _______ .

4. Let E be the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$. For any three distinct points P, Q and Q' on E, let $M(P, Q)$ be the mid-point of the line segment joining P and Q, and $M(P, Q')$ be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between $M(P, Q)$ and $M(P, Q')$, as P, Q and Q' vary on E, is _________ .

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Archive - JEE Main & Advanced

Conic Sections	Class - XI | Mathematics

JEE Main 2022

1. A particle is moving in the xy-plane along a curve C passing through the point (3, 3). The tangent to the curve C at the point P meets the x-axis at Q. If the y-axis bisects the segment PQ, then C is a parabola with:

(A) length of latus rectum 3 **(B)** length of latus rectum 6

(C) focus $\left(\frac{4}{3}, 0\right)$ **(D)** focus $\left(0, \frac{3}{4}\right)$

2. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1, a>2,$ having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellipse is:

(A) $\frac{\sqrt{3}}{2}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{\sqrt{2}}$ **(D)** $\frac{\sqrt{3}}{4}$

3. Let the hyperbola $H:\frac{x^2}{a^2}-y^2=1$ and the ellipse $E:3x^2+4y^2=12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12\left(e_H^2+e_E^2\right)$ is equal to ________.

4. Let P_1 be parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line $x+2y=6$. Then the directrix of P_2 is $x+2y=$ ______.

5. Let $x^2+y^2+Ax+By+C=0$ be a circle passing through (0, 6) and touching the parabola $y=x^2$ at (2, 4). Then $A+C$ is equal to:

(A) 16 **(B)** 88/5 **(C)** 72 **(D)** –8

6. Let $\lambda x-2y=\mu$ be a tangent to the hyperbola $a^2x^2-y^2=b^2$. Then $\left(\frac{\lambda}{a}\right)^2-\left(\frac{\mu}{b}\right)^2$ is equal to:

(A) –2 **(B)** –4 **(C)** 2 **(D)** 4

7. If two tangents drawn from a point (α, β) lying on the ellipse $25x^2+4y^2=1$ to the parabola $y^2=4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha+5)^2+(16\beta^2+50)^2$ equals ______________.

8. If the line $y=4+kx, k>0,$ is the tangent to the parabola $y=x-x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is:

(A) $\frac{3}{2}$ **(B)** $\frac{26}{9}$ **(C)** $\frac{5}{2}$ **(D)** $\frac{23}{6}$

9. The line $y=x+1$ meets the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to:

(A) 20 **(B)** 12 **(C)** 11 **(D)** 8

10. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to __________.

11. If $y=m_1x+c_1$ and $y=m_2x+c_2, m_1 \neq m_2$ are two common tangents of circle $x^2+y^2=2$ and parabola $y^2=x$, then the value of $8|m_1m_2|$ is equal to:

(A) $3+4\sqrt{2}$ **(B)** $-5+6\sqrt{2}$ **(C)** $-4+3\sqrt{2}$ **(D)** $7+6\sqrt{2}$

12. Let $x=2t, y=\frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that $SA \perp BA$, where A is any point on the conic. If k is the ordinate of the centroid of ΔSAB, then $\lim_{t \to 1} k$ is equal to:

(A) $\frac{17}{18}$ **(B)** $\frac{19}{18}$ **(C)** $\frac{11}{18}$ **(D)** $\frac{13}{18}$

13. If m is the slope of a common tangent to the curves $\frac{x^2}{16}+\frac{y^2}{9}=1$ and $x^2+y^2=12$, then $12m^2$ is equal to:

(A) 6 **(B)** 9 **(C)** 10 **(D)** 12

14. The locus of the mid point of the line segment joining the point $(4, 3)$ and the points on the ellipse $x^2+2y^2=4$ is an ellipse with eccentricity:

(A) $\frac{\sqrt{3}}{2}$ **(B)** $\frac{1}{2\sqrt{2}}$ **(C)** $\frac{1}{\sqrt{2}}$ **(D)** $\frac{1}{2}$

15. The normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{9}=1$ at the point $(8, 3\sqrt{3})$ on it passes through the point:

(A) $(15, -2\sqrt{3})$ **(B)** $(9, 2\sqrt{3})$ **(C)** $(-1, 9\sqrt{3})$ **(D)** $(-1, 6\sqrt{3})$

16. Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ and let L_2 be the line passing through the origin and perpendicular to L_1. If the locus of the point of intersection of L_1 and L_2 is $(x^2+y^2)^2=\alpha x^2+\beta y^2$, then $\alpha+\beta$ is equal to _______.

17. Let the normal at the point P on the parabola $y^2=6x$ pass through the point (5, –8). If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is:

(A) –3 **(B)** $-\frac{9}{4}$ **(C)** $-\frac{5}{2}$ **(D)** –2

18. Let the common tangents to the curves $4(x^2+y^2)=9$ and $y^2=4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum of this ellipse, then $\frac{l}{e^2}$ is equal to____________.

19. If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x+y-29=0$, is $x^2+ay^2+bxy+cx+dy+k=0$, then $a+b+c+d+k$ is equal to:

(A) 575 **(B)** – 575 **(C)** 576 **(D)** –576

20. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b,$ be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then a^2+b^2 is equal to:

(A) 29 **(B)** 31 **(C)** 32 **(D)** 34

21. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2=2x$ and touches the parabola $y=\left(x-\frac{1}{4}\right)^2+\alpha,$ where $\alpha>0$. Then $(4\alpha-8)^2$ is equal to _______.

22. Let $a>0, b>0$. Let e and l respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Let e' and l' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2=\frac{11}{14}l$ and $(e')^2=\frac{11}{8}l'$, then the value of $77a+44b$ is equal to:

(A) 100 **(B)** 110 **(C)** 120 **(D)** 130

23. If vertex of a parabola is $(2,-1)$ and the equation of its directrix is $4x-3y=21$, then the length of its latus rectum is:

(A) 2 **(B)** 8 **(C)** 12 **(D)** 16

24. Let the eccentricity of the hyperbola $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$, If $y=2x+c$ is a tangent to the hyperbola *H*. then the value of c^2 is equal to:

(A) 18 **(B)** 20 **(C)** 24 **(D)** 32

25. Let $P:y^2=4ax, a>0$ be a parabola with focus *S*. Let the tangents to the parabola *P* make an angle of $\frac{\pi}{4}$ with the line $y=3x+5$ touch the parabola *P* at *A* and *B*. Then the value of *a* for which *A*, *B* and *S* are collinear is:

(A) 8 only **(B)** 2 only **(C)** $\frac{1}{4}$ only **(D)** Any $a>0$

26. Let *PQ* be a focal chord of the parabola $y^2=4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3,0)$. Let the line segment *PQ* be also a focal chord of the ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a^2>b^2$. If *e* is the eccentricity of the ellipse *E*, then the value of $\frac{1}{e^2}$ is equal to:

(A) $1+\sqrt{2}$ **(B)** $3+2\sqrt{2}$ **(C)** $1+2\sqrt{3}$ **(D)** $4+5\sqrt{3}$

27. Let $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, a>0, b>0$, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4\left(2\sqrt{2}+\sqrt{14}\right)$. If the eccentricity *H* is $\frac{\sqrt{11}}{2}$, then the value of a^2+b^2 is equal to _________.

28. If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the *x*-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the *y*-axis, then the eccentricity of the ellipse is:

(A) $\frac{5}{7}$ **(B)** $\frac{2\sqrt{6}}{7}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{2\sqrt{5}}{7}$

29. The tangents at the points $A(1,3)$ and $B(1,-1)$ on the parabola $y^2-2x-2y=1$ meet at the point *P*. Then the area (in unit²) of the triangle PAB is:

(A) 4 **(B)** 6 **(C)** 7 **(D)** 8

30. Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is:

(A) $\frac{32}{9}$ **(B)** $\frac{18}{5}$ **(C)** $\frac{27}{4}$ **(D)** $\frac{27}{10}$

31. Let $P(a, b)$ be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of a and B be the product of all possible values of b. Then the value of $A + B$ is equal to:

(A) 0 **(B)** 25 **(C)** 40 **(D)** 65

32. An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H, respectively. Let the product of the eccentricities of E and H be $1/2$. If l is the length of the latus rectum of the ellipse E, then the value of $113l$ is equal to ______.

33. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4, and l is the length of its major axis, then $\lambda + l$ is equal to ______.

34. If the tangents drawn at the points P and Q on the parabola $y^2 = 2x - 3$ intersect at the point $R(0,1)$, then the orthocenter of the triangle PQR is:

(A) (0, 1) **(B)** (2, –1) **(C)** (6, 3) **(D)** (2, 1)

35. For the hyperbola $H: x^2 - y^2 = 1$ and the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b > 0$, let the

(1) eccentricity of E be reciprocal of the eccentricity of H, and

(2) the line $y = \sqrt{\frac{5}{2}}x + K$ be a common tangent of E and H.

Then $4\left(a^2 + b^2\right)$ is equal to __________.

36. Let the equation of two diameters of a circle $x^2 + y^2 - 2x + 2fy + 1 = 0$ be $2px - y = 1$ and $2x + py = 4p$. Then the slope $m \in (0, \infty)$ of the tangent to the hyperbola $3x^2 - y^2 = 3$ passing through the centre of the circle is equal to_____.

37. The sum of diameters of the circles that touch (i) the parabola $75x^2 = 64(5y - 3)$ at the point $\left(\frac{8}{5}, \frac{6}{5}\right)$ and (ii) the y-axis, is equal to _____.

38. Let the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ pass through the point $\left(2\sqrt{2}, -2\sqrt{2}\right)$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola

(A) $\left(2\sqrt{3}, 3\sqrt{2}\right)$ **(B)** $\left(3\sqrt{3}, -6\sqrt{2}\right)$ **(C)** $\left(\sqrt{3}, -\sqrt{6}\right)$ **(D)** $\left(3\sqrt{6}, 6\sqrt{2}\right)$

39. Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ meet at the point $R\left(\sqrt{2}, 2\sqrt{2}-2\right)$. If S is the focus of the ellipse on its negative major axis, then SP^2+SQ^2 is equal to _________.

40. Two tangent lines l_1 *and* l_2 are drawn from the point (2, 0) to the parabola $2y^2=-x$. If the lines l_1 *and* l_2 are also tangent to the circle $(x-5)^2+y^2=r$, then $17r$ is equal to ____________.

41. Let a line L pass through the point of intersection of the lines $bx+10y-8=0$ and $2x-3y=0, b\in R-\left\{\frac{4}{3}\right\}$. If the line L also passes through the point (1, 1) and touches the circle $17\left(x^2+y^2\right)=16$, then the eccentricity of the ellipse $\frac{x^2}{5}+\frac{y^2}{b^2}=1$ is:

(A) $\frac{2}{\sqrt{5}}$ **(B)** $\sqrt{\frac{3}{5}}$ **(C)** $\frac{1}{\sqrt{5}}$ **(D)** $\sqrt{\frac{2}{5}}$

42. Let the focal chord of the parabola $P: y^2=4x$ along the line $L: y=mx+c, m>0$ meet the parabola at the points M and N. Let the line L be a tangent to the hyperbola $H: x^2-y^2=4$. If O is the vertex of P and F is the focus of H on the positive x-axis, then the area of the quadrilateral $OMFN$ is:

(A) $2\sqrt{6}$ **(B)** $2\sqrt{14}$ **(C)** $4\sqrt{6}$ **(D)** $4\sqrt{14}$

43. The acute angle between the pair of tangents drawn to the ellipse $2x^2+3y^2=5$ from the point (1, 3) is:

(A) $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$ **(B)** $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

(C) $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$ **(D)** $\tan^{-1}\left(\frac{3+8\sqrt{5}}{35}\right)$

44. The equation of a common tangent to the parabolas $y=x^2$ and $y=-(x-2)^2$ is:

(A) $y=4(x-2)$ **(B)** $y=4(x-1)$ **(C)** $y=4(x+1)$ **(D)** $y=4(x+2)$

45. If the line $x-1=0$ is a directrix of the hyperbola $kx^2-y^2=6$, then the hyperbola passes through the point:

(A) $\left(-2\sqrt{5}, 6\right)$ **(B)** $\left(-\sqrt{5}, 3\right)$ **(C)** $\left(\sqrt{5}, -2\right)$ **(D)** $\left(2\sqrt{5}, 3\sqrt{6}\right)$

46. If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is $x+y=a$, is 16, then $|a|$ is equal to:

(A) $2\sqrt{2}$ **(B)** $2\sqrt{3}$ **(C)** $4\sqrt{2}$ **(D)** 4

47. A common tangent T to the curves $C_1 : \frac{x^2}{4} + \frac{y^2}{9} = 1$ and $C_2 : \frac{x^2}{42} - \frac{y^2}{143} = 1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2), then $|2x_1 + x_2|$ is equal to _____

48. Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha + 4, \beta + 4)$ does **NOT** pass through the point:

(A) (25, 10) **(B)** (20, 12) **(C)** (30, 8) **(D)** (15, 13)

49. Let the function $f(x) = 2x^2 - \log_e x, x > 0,$ be decreasing in $(0, a)$ and increasing in $(a, 4)$. A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point $(8a, 8a - 1)$ but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1,$ then $\alpha + \beta$ is equal to______.

Archive - JEE Main & Advanced

Inverse Trigonometric Functions	Class - XII | Mathematics

JEE Main 2021

1. $\lim_{n \to \infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to _______.

2. $\operatorname{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to:

(A) $\frac{65}{56}$ **(B)** $\frac{56}{33}$ **(C)** $\frac{65}{33}$ **(D)** $\frac{75}{56}$

3. If $\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}$; $0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is :

(A) $\frac{1-y^2}{1+y^2}$ **(B)** $\frac{1-y^2}{2y}$ **(C)** $1-y^2$ **(D)** $\frac{1-y^2}{y\sqrt{y}}$

4. If $0 < a, b < 1$, and $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$, then the value of $(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots$ is :

(A) e **(B)** $e^2 - 1$ **(C)** $\log_e\left(\frac{e}{2}\right)$ **(D)** $\log_e 2$

5. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is:

(A) $\sqrt{7} - 1$ **(B)** $2\sqrt{2} - 1$ **(C)** $\frac{1}{\sqrt{7}}$ **(D)** $\frac{1}{2\sqrt{2}}$

6. The number of solutions of the equation $\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$, for $x \in [-1, 1]$ and $[x]$ denotes the greatest integer less than or equal to x, is :

(A) 2 **(B)** 0 **(C)** Infinite **(D)** 4

7. The real values function $f(x) = \frac{\cos ec^1 x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to:

(A) all reals except the interval [–1, 1] **(B)** all non-integers except the interval [–1, 1]

(C) all integers except 0, –1, 1 **(D)** all reals except integers

8. Given that the inverse trigonometric functions take principal values only. Then the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1}x$ is equal to :

(A) 0 **(B)** 3 **(C)** 1 **(D)** 2

9. If $\cot^{-1}(\alpha) = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \ldots\ldots$ upto 100 terms, then α is:

(A) 1.00 **(B)** 1.02 **(C)** 1.03 **(D)** 1.01

10. The sum of possible values of x for $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$ is:

(A) $-\frac{30}{4}$ **(B)** $-\frac{32}{4}$ **(C)** $-\frac{33}{4}$ **(D)** $-\frac{31}{4}$

11. Let $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$. Then $\lim_{k\to\infty}S_k$ is equal to :

(A) $\cot^{-1}\left(\frac{3}{2}\right)$ **(B)** $\frac{\pi}{2}$ **(C)** $\tan^{-1}\left(\frac{3}{2}\right)$ **(D)** $\tan^{-1}(3)$

12. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$ is :

(A) 1 **(B)** 2 **(C)** 0 **(D)** 4

13. The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to:

(A) $\frac{-291}{76}$ **(B)** $\frac{220}{21}$ **(C)** $\frac{151}{63}$ **(D)** $\frac{-181}{69}$

14. If the domain of the function $f(x)=\frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha,\beta]$, then $\alpha+\beta$ is equal to:

(A) 2 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** $\frac{3}{2}$

15. The domain of the function $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$ is:

(A) $\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$ **(B)** $\left[0,\frac{1}{2}\right]$ **(C)** $[-2,0]\cup\left[\frac{1}{4},\frac{1}{2}\right]$ **(D)** $\left[0,\frac{1}{4}\right]$

16. If $(\sin^{-1}x)^2-(\cos^{-1}x)^2=a;\ 0<x<1,\ a\neq 0,$ then the value of $2x^2-1$ is:

(A) $\cos\left(\frac{2a}{\pi}\right)$ **(B)** $\sin\left(\frac{4a}{\pi}\right)$ **(C)** $\sin\left(\frac{2a}{\pi}\right)$ **(D)** $\cos\left(\frac{4a}{\pi}\right)$

17. If $\sum_{r=1}^{50}\tan^{-1}\frac{1}{2r^2}=p,$ then the value of $\tan p$ is:

(A) $\frac{51}{50}$ **(B)** $\frac{50}{51}$ **(C)** $\frac{101}{102}$ **(D)** 100

18. Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right),\ 0<x<1.$ Then:

(A) $(1-x)^2 f'(x)+2(f(x))^{2=0}$ **(B)** $(1-x)^2 f'(x)-2(f(x))^2 0$

(C) $(1+x)^2 f'(x)+2(f(x))^2=0$ **(D)** $(1+x)^2 f'(x)-2(f(x))^2=0$

19. $\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6))-\tan^{-1}(\tan(12))$ is equal to:

(The inverse trigonometric functions take the principal values)

(A) $4\pi-11$ **(B)** $3\pi+1$ **(C)** $4\pi-9$ **(D)** $3\pi-11$

JEE Advanced 2021

1. For any positive integer n, let $S_n : (0,\infty) \to R$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right)$$

Where for any $x \in R$, $\cot^{-1}(x) \in (0,\pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then which of the following statements is (are) TRUE?

(A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)$, for all $x > 0$

(B) $\lim_{n\to\infty} \cot(S_n(x)) = x$, for all $x > 0$

(C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0,\infty)$

(D) $\tan(S_n(x)) \le \frac{1}{2}$, for all $n \ge 1$ and $x > 0$

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Archive - JEE Main & Advanced

Inverse Trigonometric Functions	Class - XII | Mathematics

JEE Main 2022

1. $\tan\left(2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right)$ is equal to:

(A) 1 **(B)** 2 **(C)** $\frac{1}{4}$ **(D)** $\frac{5}{4}$

2. If $0<x<\frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1}x}{\alpha}=\frac{\cos^{-1}x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is:

(A) $4\sqrt{(1-x^2)}(1-2x^2)$ **(B)** $4x\sqrt{(1-x^2)}(1-2x^2)$

(C) $2x\sqrt{(1-x^2)}(1-4x^2)$ **(D)** $4\sqrt{(1-x^2)}(1-4x^2)$

3. If the maximum value of a, for which the function $f_a(x)=\tan^{-1}2x-3ax+7$ is non-decreasing in $\left(-\frac{\pi}{6},\frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to:

(A) $8-\frac{9\pi}{4(9+\pi^2)}$ **(B)** $8-\frac{4\pi}{9(4+\pi^2)}$ **(C)** $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$ **(D)** $8-\frac{\pi}{4}$

4. Let $x*y=x^2+y^3$ and $(x*1)*1=x*(1*1)$. Then a value of $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)$ is:

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{6}$

5. If $y=\tan^{-1}\left(\sec x^3-\tan x^3\right)$, $\frac{\pi}{2}<x^3<\frac{3\pi}{2}$, then:

(A) $xy''+2y'=0$ **(B)** $x^2y''-6y+\frac{3\pi}{2}=0$

(C) $x^2y''-6y+3\pi=0$ **(D)** $xy''-4y'=0$

6. The set of all values of k for which $(\tan^{-1}x)^3+(\cot^{-1}x)^3=k\pi^3$, $x\in R$, is the interval:

(A) $\left[\frac{1}{32},\frac{7}{8}\right)$ **(B)** $\left(\frac{1}{24},\frac{13}{16}\right)$ **(C)** $\left[\frac{1}{48},\frac{13}{16}\right]$ **(D)** $\left[\frac{1}{32},\frac{9}{8}\right)$

7. The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to:

(A) $-\frac{\pi}{4}$ **(B)** $-\frac{\pi}{8}$ **(C)** $-\frac{5\pi}{12}$ **(D)** $-\frac{4\pi}{9}$

8. If the inverse trigonometric functions take principal values, then

$$\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$$ is equal to:

(A) 0 **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{6}$

9. Let $f(x)=2\cos^{-1}x+4\cot^{-1}x-3x^2-2x+10, x\in[-1,1]$. If $[a,b]$ is the range of the function f, then $4a-b$ is equal to:

(A) 11 **(B)** $11-\pi$ **(C)** $11+\pi$ **(D)** $15-\pi$

10. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is:

(A) $\frac{26}{25}$ **(B)** $\frac{25}{26}$ **(C)** $\frac{50}{51}$ **(D)** $\frac{52}{51}$

11. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to:

(A) $\frac{11\pi}{12}$ **(B)** $\frac{17\pi}{12}$ **(C)** $\frac{31\pi}{12}$ **(D)** $\frac{-3\pi}{4}$

12. The value of $\lim_{n\to\infty} 6\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ is equal to:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6

13. $50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right)+2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$ is equal to ________.

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Archive - JEE Main & Advanced

Functions	Class - XII | Mathematics

JEE Main 2021

1. Let $f: R \to R$ be defined as $f(x) = 2x - 1$ and $g: R - \{1\} \to R$ be defined as $g(x) = \dfrac{x - \frac{1}{2}}{x - 1}$. Then the composition function $f(g(x))$ is:

(A) onto but not one-one
(B) both one-one and onto
(C) neither one-one nor onto
(D) one-one but not onto

2. If $a + \alpha = 1, b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, x \neq 0,$ then the value of the expression $\dfrac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$ is __________ .

3. Let $f, g: N \to N$ such that $f(n+1) = f(n) + f(1)\ \forall\ n \in N$ and g be any arbitrary function. Which of the following statements is NOT true ?

(A) If f is onto, then $f(n) = n\ \forall\ n \in N$
(B) If g is onto, then fog is one-one
(C) If fog is one-one, then g is one-one
(D) f is one-one

4. A function $f(x)$ is given by $f(x) = \dfrac{5^x}{5^x + 5}$, then the sum of the series

$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + ... + f\left(\frac{39}{20}\right)$ is equal to:

(A) $\frac{29}{2}$
(B) $\frac{19}{2}$
(C) $\frac{49}{2}$
(D) $\frac{39}{2}$

5. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is _______.

6. Let $f(x) = \sin^{-1} x$ and $g(x) = \dfrac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim\limits_{x \to 2} g(x)$, then the domain of the function fog is:

(A) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$
(B) $(-\infty, -2] \cup [-1, \infty)$
(C) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$
(D) $(-\infty, -1] \cup [2, \infty)$

7. If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions: $f+g,\ f-g,\ f/g,\ g/f,\ g-f$ where $(f \pm g)(x) = f(x) \pm g(x), (f/g)(x) = \dfrac{f(x)}{g(x)}$

(A) $0 < x \le 1$
(B) $0 < x < 1$
(C) $0 \le x \le 1$
(D) $0 \le x < 1$

8. The inverse of $y = 5^{\log x}$ is:

(A) $x = 5^{\log y}$
(B) $x = y^{\log 5}$
(C) $x = 5^{\frac{1}{\log y}}$
(D) $x = y^{\frac{1}{\log 5}}$

9. Let $f: R - \{3\} \to R - \{1\}$ defined by $f(x) = \dfrac{x-2}{x-3}$.

Let $g: R \to R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \dfrac{13}{2}$ is equal to:

(A) 2
(B) 7
(C) 3
(D) 5

10. The number of elements in the set $\{x \in R : (|x|-3)|x+4| = 6\}$ is equal to :

(A) 1 **(B)** 3 **(C)** 2 **(D)** 4

11. Consider functions $f : A \to B$ and $g : B \to C (A, B, C \subseteq R)$ such that $(gof)^{-1}$ exists, then:

(A) *f* is one-one and *g* is into **(B)** *f* and *g* both are onto

(C) *f* is onto and *g* is one-one **(D)** *f* and *g* both are one-one

12. Let $g : N \to N$ be defined as

$g(3n+1) = 3n+2,$
$g(3n+2) = 3n+3,$
$g(3n+3) = 3n+1,$ for all $n \geq 0$.

Then which of the following statements is true?

(A) There exists an onto function $f : N \to N$ such that $fog = f$

(B) There exists a one-one function $f : N \to N$ such that $fog = f$

(C) $gogog = g$

(D) There exists a function $f : N \to N$ such that $gof = f$

13. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^{x} + 1 = 0$ is:

(A) 4 **(B)** 1 **(C)** 6 **(D)** 2

14. Let $[x]$ denotes the greatest integer $\leq x$, where $x \in R$. If the domain of the real valued function

$f(x) = \sqrt{\dfrac{|[x]| - 2}{|[x]| - 3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty), a < b < c$, then the value of $a + b + c$ is:

(A) 1 **(B)** -3 **(C)** -2 **(D)** 8

15. Let $f : R - \left\{\dfrac{\alpha}{6}\right\} \to R$ be defined by $f(x) = \dfrac{5x+3}{6x-\alpha}$. Then the value of α for which $(f0f)(x) = x$, for all $x \in R - \left\{\dfrac{\alpha}{6}\right\}$, is:

(A) 5 **(B)** 8 **(C)** 6 **(D)** No such α exists

16. Let $f : N \to N$ be a function such that $f(m+n) = f(m) + f(n)$ for every $m, n \in N$. If $f(6) = 18$, then $f(2) \cdot f(3)$ is equal to:

(A) 6 **(B)** 54 **(C)** 18 **(D)** 36

17. If $A = \{x \in \mathbf{R} : |x-2| > 1\}, B = \{x \in \mathbf{R} : \sqrt{x^2 - 3} > 1\}, C = \{x \in \mathbf{R} : |x-4| \geq 2\}$ and **Z** is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is ___________ .

18. The domain of the function $\text{cosec}^{-1}\left(\dfrac{1+x}{x}\right)$ is:

(A) $\left(-\dfrac{1}{2}, \infty\right) - \{0\}$ **(B)** $\left[-\dfrac{1}{2}, 0\right) \cup [1, \infty)$

(C) $\left(-1, -\dfrac{1}{2}\right] \cup (0, \infty)$ **(D)** $\left[-\dfrac{1}{2}, \infty\right) - \{0\}$

19. The range of the function $f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\dfrac{3\pi}{4} + x\right) + \cos\left(\dfrac{\pi}{4} + x\right) + \cos\left(\dfrac{\pi}{4} - x\right) - \cos\left(\dfrac{3\pi}{4} - x\right)\right)$ is:

(A) $\left[\dfrac{1}{\sqrt{5}}, \sqrt{5}\right]$ **(B)** $[0, 2]$ **(C)** $[-2, 2]$ **(D)** $(0, \sqrt{5})$

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Functions	Class - XII | Mathematics

JEE Main 2022

1. The number of functions *f*, from the set $A = \{x \in N : x^2 - 10x + 9 \le 0\}$ to the set $B = \{n^2 : n \in N\}$ such that $f(x) \le (x-3)^2 + 1$, for every $x \in A$, is _______.

2. The domain of the function $f(x) = \sin^{-1}\left[2x^2 - 3\right] + \log_2\left(\log_{\frac{1}{2}}\left(x^2 - 5x + 5\right)\right)$, where $[t]$ is the greatest integer functions, is:

(A) $\left(-\sqrt{\frac{5}{2}}, \frac{5-\sqrt{5}}{2}\right)$ **(B)** $\left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$

(C) $\left(1, \frac{5-\sqrt{5}}{2}\right)$ **(D)** $\left(1, \frac{5+\sqrt{5}}{2}\right)$

3. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. Then the number of elements in the set $\{C \subseteq A : C \cap B \ne \phi\}$ is _______.

4. Let $S = \{4, 6, 9\}$ and $T = \{9, 10, 11, ..., 1000\}$. If $A = \{a_1 + a_2 + ... + a_k : k \in N, a_1, a_2, a_3, ..., a_k \in S\}$, then the sum of all the elements in the set $T - A$ is equal to_________.

5. Let *R* be a relation from the set $\{1, 2, 3, ..., 60\}$ to itself such that $R = \{(a, b) : b = pq$, where $p, q \ge 3$ are prime numbers}. Then, the number of elements in *R* is:

(A) 600 **(B)** 660 **(C)** 540 **(D)** 720

6. The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ is:

(A) $[1, \infty)$ **(B)** $[-1, 2]$ **(C)** $[-1, \infty)$ **(D)** $(-\infty, 2]$

7. The number of elements in the set $S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}\right\}$ is:

(A) 1 **(B)** 3 **(C)** 0 **(D)** infinite

8. Let $x, y > 0$. If $x^3 y^2 = 2^{15}$, then the least value of $3x + 2y$ is:

(A) 30 **(B)** 32 **(C)** 36 **(D)** 40

9. Let $f(x) = \begin{cases} \dfrac{\sin(x - [x])}{x - [x]} & , \; x \in (-2, -1) \\ \max\{2x, 3[|x|]\} & , \; |x| < 1 \\ 1 & , \; otherwise \end{cases}$

where $[t]$ denotes greatest integer $\le t$. If *m* is the number of points where *f* is not continuous and *n* is the number of points where *f* is not differentiable, then the ordered pair (*m*, *n*) is:

(A) (3, 3) **(B)** (2, 4) **(C)** (2, 3) **(D)** (3, 4)

10. Let $S=\left\{\begin{pmatrix}-1 & a\\ 0 & b\end{pmatrix}; a, b\in\{1, 2, 3, ...100\}\right\}$ and let $T_n=\{A\in S: A^{n(n+1)}=I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is _______.

11. The domain of the function $f(x)=\dfrac{\cos^{-1}\left(\dfrac{x^2-5x+6}{x^2-9}\right)}{\log_e(x^2-3x+2)}$ is:

(A) $(-\infty, 1)\cup(2, \infty)$ **(B)** $(2, \infty)$

(C) $\left[-\dfrac{1}{2}, 1\right)\cup(2, \infty)$ **(D)** $\left[-\dfrac{1}{2}, 1\right)\cup(2, \infty)-\left\{\dfrac{3+\sqrt{5}}{2}, \dfrac{3-\sqrt{5}}{2}\right\}$

12. The number of one-one functions $f:(a, b, c, d)\to\{0, 1, 2, ..., 10\}$ such that $2f(a)-f(b)+3f(c)+f(d)=0$ is _________.

13. Let $\underset{0\le x\le 2}{Max}\left\{\dfrac{9-x^2}{5-x}\right\}=\alpha$ and $\underset{0\le x\le 2}{Min}\left\{\dfrac{9-x^2}{5-x}\right\}=\beta$

If $\displaystyle\int_{\beta-\frac{8}{3}}^{2\alpha-1} Max\left\{\dfrac{9-x^2}{5-x}, x\right\}dx=\alpha_1+\alpha_2\log_e\left(\dfrac{8}{15}\right)$, then $\alpha_1+\alpha_2$ is equal to ______________.

14. Let $A=\{x\in R: |x+1|<2\}$ and $B=\{x\in R: |x-1|\ge 2\}$. Then which one of the following statements is **NOT** true?

(A) $A-B=(-1, 1)$ **(B)** $B-A=R-(-3, 1)$

(C) $A\cap B=(-3, -1]$ **(D)** $A\cup B=R-[1, 3)$

15. Let $f:N\to R$ be a function such that $f(x+y)=2f(x)f(y)$ for natural numbers x and y. If $f(1)=2$, then the value of α for which $\displaystyle\sum_{k=1}^{10} f(\alpha+k)=\frac{512}{3}(2^{20}-1)$ holds, is:

(A) 2 **(B)** 3 **(C)** 4 **(D)** 6

16. Let $f:R\to R$ be defined as $f(x)=x^3+x-5$. If $g(x)$ is a function such that $f(g(x))=x, \forall 'x'\in R$ then $g'(63)$ is equal to ______________.

(A) $\dfrac{1}{49}$ **(B)** $\dfrac{3}{49}$ **(C)** $\dfrac{43}{49}$ **(D)** $\dfrac{91}{49}$

17. Let $f:R\to R$ and $g:R\to R$ be two functions defined by $f(x)=\log_e(x^2+1)-e^{-x}+1$ and $g(x)=\dfrac{1-2e^{2x}}{e^x}$.

Then, for which of the following range of α, the inequality $f\left(g\left(\dfrac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\dfrac{5}{3}\right)\right)$ holds?

(A) (2, 3) **(B)** (–2, – 1) **(C)** (1, 2) **(D)** (–1, 1)

18. Let $f:R\to R$ be a function defined by $f(x)=\left(2\left(1-\dfrac{x^{25}}{2}\right)(2+x^{25})\right)^{\frac{1}{50}}$. If the function $g(x)=f(f(f(x)))+f(f(x))$, then the greatest integer less than or equal to $g(1)$ is________.

19. Let $f: R \to R$ be defined as $f(x) = x - 1$ and $g: R - \{1, -1\} \to R$ be defined as $g(x) = \frac{x^2}{x^2 - 1}$. Then the function *fog* is:

(A) one-one but not onto **(B)** onto but not one-one

(C) both one-one and onto **(D)** neither one-one nor onto

20. Let $f: R \to R$ satisfies $f(x+y) = 2^x f(y) + 4^y f(x), \forall x, y \in R$. If $f(2) = 3$, then $14 \cdot \frac{f'(4)}{f'(2)}$ is equal to_______.

21. Let $f(x) = \frac{x-1}{x+1}, x \in R - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^n(x))$ for all $n \in N$, then $f^6(6) + f^7(7)$ is equal to:

(A) $\frac{7}{6}$ **(B)** $-\frac{3}{2}$ **(C)** $\frac{7}{12}$ **(D)** $-\frac{11}{12}$

22. Lert $f, g: R \to R$ be two real valued functions defined as $f(x) = \begin{cases} -|x+3| & , x < 0 \\ e^x & , x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x^2 + k_1 x & , x < 0 \\ 4x + k_2 & , x \geq 0 \end{cases}$, where k_1 and k_2 are real constants. If (gof) is differentiable at $x = 0$, then (gof) (–4) + (gof) (4) is equal to:

(A) $4(e^4 + 1)$ **(B)** $2(2e^4 + 1)$ **(C)** $4e^4$ **(D)** $2(2e^4 - 1)$

23. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Define $f: S \to S$ as

$$f(n) = \begin{cases} 2n, & if\ n = 1, 2, 3, 4, 5 \\ 2n - 11, & if\ n = 6, 7, 8, 9, 10 \end{cases}.$$

Let $g: S \to S$ be a function such that $fog(n) = \begin{cases} n+1, & if\ n\ is\ odd \\ n-1, & if\ n\ is\ even \end{cases}$,

Then $g(10)\left(g(1) + g(2) + g(3) + g(4) + g(5)\right)$ is equal to _______.

24. Let $f: R \to R$ be a function defined by $f(x) = \frac{2e^{2x}}{e^{2x} + e}$.

Then $f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + ... + f\left(\frac{99}{100}\right)$ is equal to ____.

25. Let $R_1 = \{(a, b) \in N \times N : |a - b| \leq 13\}$ and $R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$. Then on N:

(A) Both R_1 and R_2 are equivalence relations

(B) Neither R_1 nor R_2 is an equivalence relation

(C) R_1 is an equivalence relation but R_2 is not

(D) R_1 is an equivalence relation but R_1 is not

26. Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f: S \times S \to S : f$ is onto and $f(a, b) = f(b, a) \geq a \forall (a, b) \in S \times S\}$ is ______________.

27. Let a function $f: N \to N$ be defined by $f(n) = \begin{bmatrix} 2n, & n = 2, 4, 6, 8, ... \\ n - 1, & n = 3, 7, 11, 15, ... \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, ... \end{bmatrix}$ then, f is:

(A) one-one but not onto **(B)** onto but not one-one

(C) neither one-one nor onto **(D)** one-one and onto

28. Let R_1 and R_2 be relations on the set {1, 2, ... 50} such that

$R_1 = \left\{\left(p, p^n\right) : p \text{ is a prime and } n \geq 0 \text{ is an integer}\right\}$ *and*

$R_2 = \left\{\left(p, p^n\right) : p \text{ is a prime and } n = 0 \text{ or } 1\right\}$.

Then, the number of elements in $R_1 - R_2$ is _______.

29. Let $A = \{1, a_1, a_2 .... a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < ... < a_{18} < 77$.

Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + ... + a_{18}$ is equal to ______.

30. Let $f(x)$ and $g(x)$ be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$, and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is ______.

31. Let a set $A = A_1 \cup A_2 \cup ... \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$, $1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i, 1 \leq i \leq k\}$. Then, R is:

(A) Reflexive, symmetric but not transitive

(B) Reflexive, transitive but not symmetric

(C) Reflexive but not symmetric and transitive

(D) An equivalence relation

32. The domain of the function $\cos^{-1}\left(\dfrac{2\sin^{-1}\left(\dfrac{1}{4x^2-1}\right)}{\pi}\right)$ is:

(A) $R - \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

(B) $\left(-\infty, -1\right] \cup \left[\frac{1}{2}, \infty\right) \cup \{0\}$

(C) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

(D) $\left(-\infty, \frac{-1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

33. Let $c, k \in R$. If $f(x) = (c+1)x^2 + (1-c^2)x + 2k$ and $f(x+y) = f(x) + f(y) - xy$, for all $x, y \in R$, then the value of $\left|2\left(f(1) + f(2) + f(3) + ... + f(20)\right)\right|$ is equal to _________.

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Archive - JEE Main & Advanced

Differential Calculus-1	Class - XII | Mathematics

JEE Main 2021

1. If $f: R \to R$ is a function defined by $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest integer function, then f is:

(A) discontinuous at all integral values of x except at $x = 1$
(B) continuous only at $x = 1$
(C) continuous for every real x
(D) discontinuous only at $x = 1$

2. $\lim_{x \to 0} \frac{\int_0^{x^2} \left(\sin\sqrt{t}\right) dt}{x^3}$ is equal to:

(A) $\frac{2}{3}$ **(B)** $\frac{3}{2}$ **(C)** 0 **(D)** $\frac{1}{15}$

3. $\lim_{n \to \infty} \left(1 + \frac{1 + \frac{1}{2} + ... + \frac{1}{n}}{n^2}\right)^n$ is equal to :

(A) 0 **(B)** $\frac{1}{e}$ **(C)** 1 **(D)** $\frac{1}{2}$

4. The number of points, at which the function $f(x) = |2x+1| - 3|x+2| + |x^2 + x - 2|, x \in R$ is not differentiable, is _______.

5. A function f is defined on $[-3, 3]$ as $f(x) = \begin{cases} \min\{|x|, 2 - x^2\} & , \quad -2 \le x \le 2 \\ [|x|] & , \quad 2 < |x| \le 3 \end{cases}$ where $[x]$ denotes the greatest integer $\le x$. The number of points, where f is not differentiable in $(-3, 3)$ is _______ .

6. If $\lim_{x \to 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)}$ exists and is equal to b, then the value of $a - 2b$ is __________ .

7. Let f be any function defined on R and let it satisfy the condition :

$$|f(x) - f(y)| \le |(x-y)^2|, \forall (x, y) \in R$$

If $f(0) = 1$, then :

(A) $f(x) = 0, \forall x \in R$ **(B)** $f(x)$ can take any value in R
(C) $f(x) > 0, \forall x \in R$ **(D)** $f(x) < 0, \forall x \in R$

8. The value of $\lim_{h \to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6} + h\right) - \cos\left(\frac{\pi}{6} + h\right)}{\sqrt{3}h(\sqrt{3}\cos h - \sin h)}\right\}$ is :

(A) $\frac{2}{3}$ **(B)** $\frac{2}{\sqrt{3}}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{3}$

9. Let $f : R \to R$ be defined as $f(x) = \begin{cases} 2\sin\left(-\dfrac{\pi x}{2}\right), & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x), & \text{if } x > 1 \end{cases}$

If $f(x)$ is continuous on R, then $a + b$ equals :

(A) -3 **(B)** -1 **(C)** 3 **(D)** 1

10. Let $f(x)$ be a differentiable function at $x = a$ with $f'(a) = 2$ and $f(a) = 4$. Then $\lim\limits_{x \to a} \dfrac{xf(a) - af(x)}{x - a}$ equals :

(A) $2a - 4$ **(B)** $2a + 4$ **(C)** $a + 4$ **(D)** $4 - 2a$

11. The value of $\lim\limits_{n \to \infty} \dfrac{[r] + [2r] + ... + [nr]}{n^2}$, where r is a non-zero number and $[r]$ denotes the greatest integer than or equal to r, is equal to :

(A) $2r$ **(B)** r **(C)** 0 **(D)** $\dfrac{r}{2}$

12. The value of the limit $\lim\limits_{\theta \to 0} \dfrac{\tan(\pi \cos^2 \theta)}{\sin(2\pi \sin^2 \theta)}$ is equal to :

(A) $\dfrac{1}{4}$ **(B)** $-\dfrac{1}{4}$ **(C)** $-\dfrac{1}{2}$ **(D)** 0

13. If $f(x) = \begin{cases} \dfrac{1}{|x|} & ; |x| \ge 1 \\ ax^2 + b; & |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively:

(A) $\dfrac{1}{2}, \dfrac{1}{2}$ **(B)** $\dfrac{1}{2}, -\dfrac{3}{2}$ **(C)** $-\dfrac{1}{2}, \dfrac{3}{2}$ **(D)** $\dfrac{5}{2}, -\dfrac{3}{2}$

14. Let $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x+1) = xf(x)$. If $g : S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to :

(A) $\dfrac{205}{144}$ **(B)** $\dfrac{197}{144}$ **(C)** $\dfrac{187}{144}$ **(D)** 1

15. If the function $f(x) = \dfrac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \dfrac{1}{k}$, then k is____.

16. The value of $\lim\limits_{x \to 0^+} \dfrac{\cos^{-1}(x - [x]^2) \cdot \sin^{-1}(x - [x]^2)}{x - x^3}$, where [x] denotes the greatest integer $\le x$ is:

(A) π **(B)** $\dfrac{\pi}{4}$ **(C)** 0 **(D)** $\dfrac{\pi}{2}$

17. Let $f : R \to R$ be a function defined as $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin 2x}{2x}, & \text{if } x < 0 \\ b, & \text{if } x = 0 \\ \dfrac{\sqrt{x + bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{if } x > 0 \end{cases}$.

If f is continuous at $x = 0$, then the value of $a + b$ is equal to:

(A) $-\dfrac{5}{2}$ **(B)** $-\dfrac{3}{2}$ **(C)** -2 **(D)** -3

18. Let $f : R \to R$ satisfy the equation $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in R$ and $f(x) \ne 0$ for any $x \in R$. If the function f is differentiable at $x = 0$ and $f'(0) = 3$, then $\lim\limits_{h \to 0} \dfrac{1}{h}(f(h - 1)$ is equal to:

19. Let the functions $f : R \to R$ and $g : R \to R$ be defined as :

$$f(x) = \begin{cases} x+2, & x<0 \\ x^2, & x \ge 0 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x^3, & x<1 \\ 3x-1, & x \ge 1 \end{cases}$$

Then, the number of points in R where $(fog)(x)$ is NOT differentiable is equal to :

(A) 0 **(B)** 3 **(C)** 2 **(D)** 1

20. If $\lim\limits_{x\to 0} \dfrac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$, then $a+b+c$ is equal to ________.

21. Let $f : [0,\infty) \to [0,3]$ be a function defined by

$$f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2+\cos x, & x > \pi \end{cases}$$

Then which of the following is true?

(A) f is continuous everywhere but not differentiable exactly at two points in $(0,\infty)$

(B) f is not continuous exactly at two points in $(0,\infty)$

(C) f is differentiable everywhere in $(0,\infty)$

(D) f is continuous everywhere but not differentiable exactly at one point in $(0,\infty)$

22. The value of $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt[8]{1-\sin x} - \sqrt[8]{1+\sin x}}\right)$ is equal to :

(A) –1 **(B)** 0 **(C)** 4 **(D)** –4

23. Let $f : [0, 3] \to R$ be defined by $f(x) = \min\{x-[x], 1+[x]-x\}$ where $[x]$ is the greater integer less than of equal to x.

Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous, and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to _____.

24. Let $f : R \to R$ be a function such that $f(2) = 4$ and $f'(2) = 1$. Then, the value of $\lim\limits_{x\to 2}\dfrac{x^2 f(2) - 4f(x)}{x-2}$ is equal to :

(A) 12 **(B)** 16 **(C)** 8 **(D)** 4

25. Let $f : \left(-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right) \to R$ be defined as $f(x) = \begin{cases} (1+|\sin x|)^{\frac{3a}{|\sin x|}}, & -\dfrac{\pi}{4} < x < 0 \\ b, & x = 0 \\ e^{\cot 4x/\cot 2x}, & 0 < x < \dfrac{\pi}{4} \end{cases}$. If f is continuous at $x = 0$, then the value of $6x + b^2$ is equal to :

(A) e **(B)** $1-e$ **(C)** $1+e$ **(D)** $e-1$

26. Consider the function $f(x) = \dfrac{P(x)}{\sin(x-2)}, x \ne 2$

$= 7 \quad , x = 2$

Where $P(x)$ is a polynomial such that $P''(x)$ is always a constant and $P(3) = 9$. If $f(x)$ is continuous at $x = 2$, then $P(5)$ is equal to ______.

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as $f(x) = \begin{cases} 3\left(1 - \dfrac{|x|}{2}\right) & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases}$

Let $g : \mathbb{R} \to \mathbb{R}$ be given by $g(x) = f(x+2) - f(x-2)$. If n and m denote the number of points in $\mathbb{R}$ where g is not continuous and not differentiable, respectively, then $n + m$ is equal to ______.

28. Let $f: R \to R$ be defined as $f(x) = \begin{cases} \dfrac{x^3}{(1-\cos 2x)^2} \log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right), & x \neq 0 \\ \alpha, & x = 0 \end{cases}$. If f is continuous at $x = 0$, then α is equal to:

(A) 0 **(B)** 3 **(C)** 2 **(D)** 1

29. If $\lim\limits_{x \to 0} \dfrac{\alpha x e^x - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10, \alpha, \beta, \gamma \in R$, then the value of $\alpha + \beta + \gamma +$ is __________ .

30. Let a function $f: R \to R$ be defined as $f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \ge 1 \end{cases}$

Where $[x]$ is the greatest integer less than or equal to x. If f is continuous on R, then $(a+b)$ is equal to:

(A) 4 **(B)** 3 **(C)** 2 **(D)** 5

31. If the value of $\lim\limits_{x \to 0} \left(2 - \cos x \sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to ______.

32. If $\alpha = \lim\limits_{x \to \pi/4} \dfrac{\tan^3 x - \tan x}{\cos\left(x + \dfrac{\pi}{4}\right)}$ and $\beta = \lim\limits_{x \to 0} (\cos x)^{\cot x}$ are the roots of the equation $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is:

(A) (1, –3) **(B)** (1, 3) **(C)** (–1, 3) **(D)** (–1, –3)

33. If α, β are the distinct roots of $x^2 + bx + c = 0$, then $\lim\limits_{x \to \beta} \dfrac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x-\beta)^2}$ is equal to :

(A) $b^2 + 4c$ **(B)** $b^2 - 4c$ **(C)** $2(b^2 + 4c)$ **(D)** $2(b^2 - 4c)$

34. Let $[t]$ denote the greatest integer less than or equal to t.
Let $f(x) = x - [x]$, $g(x) = 1 - x + [x]$, and $h(x) = \min\{f(x), g(x)\}$, $x \in [-2, 2]$.
Then h is:

(A) Continuous in [–2, 2] but not differentiable at exactly three points in (–2, 2)
(B) Not continuous at exactly four points in [–2, 2]
(C) Not continuous at exactly three points in [–2, 2]
(D) Continuous in [–2, 2] but not differentiable at more than four points in (–2, 2)

35. Let $a, b \in R, b \neq 0$. Define a function $f(x) = \begin{cases} a \sin \dfrac{\pi}{2}(x-1), & \text{for } x \le 0 \\ \dfrac{\tan 2x - \sin 2x}{bx^3}, & \text{for } x > 0 \end{cases}$.

If f is continuous at $x = 0$, then $10 - ab$ is equal to ________.

36. Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3, x \in R$. Then the natural number n for which

$\lim\limits_{x \to 1} \dfrac{x^n f(1) - f(x)}{x - 1} = 44$ is:

37. Let $[t]$ denote the greatest integer $\le t$. The number of points where the function $f(x)=[x]|x^2-1|+\sin\left(\frac{\pi}{[x]+3}\right)-[x+1], x\in(-2,2)$ is continuous is:

38. Let $f: R \to R$ be a continuous function. Then $\lim_{x\to\frac{\pi}{4}} \dfrac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)dx}{x^2-\frac{\pi^2}{16}}$ is equal to:

(A) $2f(\sqrt{2})$ **(B)** $2f(2)$ **(C)** $4f(2)$ **(D)** $f(2)$

39. The function $f(x)=x^3-6x^2+ax+b$ is such that $f(2)=f(4)=0$. Consider two statements, $(S1)$ there exists $x_1, x_2\in(2,4), x_1<x_2$, such that $f'(x_1)=-1$ and $f'(x_2)=0$. $(S2)$ there exists $x_3, x_4\in(2,4), x_3<x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3)=\sqrt{3}f(x_4)$.

(A) Both (S1) and (S2) are false **(B)** Both (S1) and (S2) are true

(C) (S1) is false and (S2) is true **(D)** (S1) is true and (S2) is false

40. If 'R' is the least value of 'a' such that the function $f(x)=x^2+ax+1$ is increasing on $[1,2]$ and 'S' is the greatest value of 'a' such that the function $f(x)=x^2+ax+1$ is decreasing on [1, 2], then the value of |R – S| is _________ .

41. $\lim_{x\to 0}\dfrac{\sin^2(\pi\cos^4 x)}{x^4}$ is equal to:

(A) $4\pi^2$ **(B)** π^2 **(C)** 4π **(D)** $2\pi^2$

42. If the function $f(x)=\begin{cases}\dfrac{1}{x}\log_e\left(\dfrac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , x<0\\ k & , x=0\\ \dfrac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1} & , x>0\end{cases}$

is continuous at $x=0$, then $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{4}{k}$ is equal to:

(A) 4 **(B)** 5 **(C)** –4 **(D)** –5

43. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a,b) is :

(A) $\left(-1,-\frac{1}{2}\right)$ **(B)** $\left(-1,\frac{1}{2}\right)$ **(C)** $\left(1,\frac{1}{2}\right)$ **(D)** $\left(1,-\frac{1}{2}\right)$

44. If $y(x)=\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x\in\left(\frac{\pi}{2},\pi\right)$, then $\dfrac{dy}{dx}$ at $x=\dfrac{5\pi}{6}$ is :

(A) $\frac{1}{2}$ **(B)** $-\frac{1}{2}$ **(C)** 0 **(D)** –1

45. Let $f: R \to R$ and $g: R \to R$ be defined as $f(x) = \begin{cases} x+a, & x<0 \\ |x-1|, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & x<0 \\ (x-1)^2+b, & x \geq 0 \end{cases}$' where a, b are non-negative real numbers. If $(gof)(x)$ is continuous for all $x \in R$, then $a+b$ is equal to __________ .

46. If $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is___________.

47. If $f(x) = \begin{cases} \int_0^x (5+|1-t|), & x>2 \\ 5x+1, & x \leq 2 \end{cases}$, then:

(A) $f(x)$ is not continuous at $x=2$

(B) $f(x)$ is everywhere differentiable

(C) $f(x)$ is continuous but not differentiable at $x=2$

(D) $f(x)$ is not differentiable at $x=1$

48. If $y^{1/4} + y^{-1/4} = 2x$, and $(x^2-1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$, then $|\alpha - \beta|$ is equal to _________.

49. If $\lim_{x \to 0} \frac{\sin^{-1}x - \tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is:

(A) $\frac{1}{2}$ **(B)** 2 **(C)** 6 **(D)** $\frac{1}{6}$

50. The function $f(x) = |x^2-2x-3| . e^{|9x^2-12x+4|}$ is not differentiable at exactly:

(A) one point **(B)** three points **(C)** four points **(D)** two points

51. If $y = y(x)$ is an implicit function of x such that $\log_e(x+y) = 4xy$, then $\frac{d^2y}{dx^2}$ at $x=0$ is equal to ________.

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Archive - JEE Main & Advanced

Differential Calculus-1	Class - XII | Mathematics

JEE Main 2022

1. If $a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2n}{n^2+k^2}$ and $f(x)=\sqrt{\frac{1-\cos x}{1+\cos x}}, x\in(0,1)$, then:

(A) $2\sqrt{2}\,f\left(\frac{a}{2}\right)=f'\left(\frac{a}{2}\right)$ **(B)** $f\left(\frac{a}{2}\right)f'\left(\frac{a}{2}\right)=\sqrt{2}$

(C) $\sqrt{2}\,f\left(\frac{a}{2}\right)=f'\left(\frac{a}{2}\right)$ **(D)** $f\left(\frac{a}{2}\right)=\sqrt{2}\,f'\left(\frac{a}{2}\right)$

2. If the function $f(x)=\begin{cases}\frac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)-\{0\}\\ k, & x=0\end{cases}$ is continuous at $x = 0$, then k is equal to:

(A) 1 **(B)** –1 **(C)** e **(D)** 0

3. If $f(x)=\begin{cases}x+a, & x\le 0\\ |x-4|, & x>0\end{cases}$ and $g(x)=\begin{cases}x+1, & x<0\\ (x-4)^2+b, & x\ge 0\end{cases}$ are continuous on R, then $(g\ of)(2)+(fog)(-2)$ is equal to:

(A) –10 **(B)** 10 **(C)** 8 **(D)** – 8

4. Let $f : R \to R$ be a continuous function such that $f(3x)-f(x)=x$. If $f(8)=7$, then $f(14)$ is equal to:

(A) 4 **(B)** 10 **(C)** 11 **(D)** 16

5. If for $p\ne q\ne 0$, the function $f(x)=\frac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$ is continuous at $x=0$, then:

(A) $7pqf(0)-1=0$ **(B)** $63q\,f(0)-p^2=0$

(C) $21q\,f(0)-p^2=0$ **(D)** $7pq\,f(0)-9=0$

6. Let $\beta=\lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)}$ for some $\alpha\in R$. Then the value of $\alpha+\beta$ is:

(A) $\frac{14}{5}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{5}{2}$ **(D)** $\frac{7}{2}$

7. The value of $\log_e 2\frac{d}{dx}(\log_{\cos x} cosec\, x)$ at $x=\frac{\pi}{4}$ is:

(A) $-2\sqrt{2}$ **(B)** $2\sqrt{2}$ **(C)** -4 **(D)** 4

8. The number of points, where the function $f : R \to R$,

$f(x)=|x-1|\cos|x-2|\sin|x-1|+(x-3)|x^2-5x+4|$, is **NOT** differentiable, is:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

9. If $\lim_{x\to 0}\dfrac{\alpha e^{x}+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\dfrac{2}{3}$, where $\alpha,\beta,\gamma\in R$, then which of the following is **NOT** correct?

(A) $\alpha^2+\beta^2+\gamma^2=6$ **(B)** $\alpha\beta+\beta\gamma+\gamma\alpha+1=0$

(C) $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$ **(D)** $\alpha^2-\beta^2+\gamma^2=4$

10. If $[t]$ denotes the greatest integer $\le t$, then the number of points, at which the function $f(x)=4\left|2x+3\right|+9\left[x+\dfrac{1}{2}\right]-12\left[x+20\right]$ is not differentiable in the open interval $(-20,20)$, is ________.

11. The number of points where the function

$$f(x)=\begin{cases}|2x^2-3x-7| & \text{if } x\le -1\\ [4x^2-1] & \text{if } -1<x<1\\ |x+1|+|x-2| & \text{if } x\ge 1\end{cases}$$

$[t]$ denotes the greatest integer $\le t$, is discontinuous is __________.

12. $\lim_{x\to\frac{\pi}{2}}\left(\tan^2 x\left(\left(2\sin^2 x+3\sin x+4\right)^{\frac{1}{2}}-\left(\sin^2 x+6\sin x+2\right)^{\frac{1}{2}}\right)\right)$ is equal to:

(A) $\dfrac{1}{12}$ **(B)** $-\dfrac{1}{18}$ **(C)** $-\dfrac{1}{12}$ **(D)** $\dfrac{1}{6}$

13. Let $f(x)=\left[2x^2+1\right]$ and $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\ge 0\end{cases}$, where $[t]$ is the greatest integer $\le t$. Then, in the open interval $(-1,1)$, the number of points where *fog* is discontinuous is equal to _________.

14. Let $f(x)$ be a polynomial function such that $f(x)+f'(x)+f''(x)=x^5+64$. Then, the value of $\lim_{x\to 1}\dfrac{f(x)}{x-1}$ is equal to:

(A) -15 **(B)** -60 **(C)** 60 **(D)** 15

15. Let $f(x)=\min\{1, 1+x\sin x\}$, $0\le x\le 2\pi$. If m is the number of points, where f is not differential and n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to:

(A) (2, 0) **(B)** (1, 0) **(C)** (1, 1) **(D)** (2, 1)

16. Let $[t]$ denote the greatest integer $\le t$ *and* $\{t\}$ denote the fractional part of t. The integer value of a for which the left hand limit of the function $f(x)=\left[1+x\right]+\dfrac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ at $x=0$ is equal to $\alpha-\dfrac{4}{3}$, is ________.

17. Let $f,g:R\to R$ be functions defined by

$$f(x)=\begin{cases}[x] & , x<0\\ |1-x| & , x\ge 0\end{cases} \text{ and } g(x)=\begin{cases}e^{x}-x & , x<0\\ (x-1)^2-1 & , x\ge 0\end{cases}$$

Where $[x]$ denote the greatest integer less than or equal to x. Then, the function *fog* is discontinuous at exactly.

(A) One point **(B)** two points **(C)** three points **(D)** four points

18. Let $f:R\to R$ be a differentiable function such that $f\left(\dfrac{\pi}{4}\right)=\sqrt{2}, f\left(\dfrac{\pi}{2}\right)=0$ and $f'\left(\dfrac{\pi}{2}\right)=1$ and let $g(x)=\int_x^{\pi/4}\left(f'(t)\sec t+\tan t\sec t\ f(t)\right)dt$ for $x\in\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right]$. Then $\lim_{x\to\left(\frac{\pi}{2}\right)^-} g(x)$ is equal to:

(A) 2 **(B)** 3 **(C)** 4 **(D)** -3

19. If $\lim_{x\to 1}\frac{\sin\left(3x^2-4x+1\right)-x^2+1}{2x^3-7x^2+ax+b}=-2$, then the value of $\left(a-b\right)$ is equal to ____________.

20. Let $f:R\to R$ be defined as

$$f\left(x\right)=\begin{bmatrix} \left[e^x\right], & x<0 \\ ae^x+[x-1], & 0\le x<1 \\ b+\left[\sin(\pi x)\right], & 1\le x<2 \\ \left[e^{-x}\right]-c, & x\ge 2 \end{bmatrix}$$

Where $a,b,c\in R$ and $[t]$ denotes greatest integer less than or equal to *t*. Then, which of the following statements is true?

(A) There exists $a,b,c\in R$ such that *f* is continuous on *R*.

(B) If *f* is discontinuous at exactly one point, then $a+b+c=1$

(C) If *f* is discontinuous at exactly one point, then $a+b+c\neq 1$

(D) *f* is discontinuous atleast two points, for any values of *a*, *b* and *c*.

21. The value of $\lim_{x\to 1}\frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to:

(A) $\frac{\pi^2}{6}$ **(B)** $\frac{\pi^2}{3}$ **(C)** $\frac{\pi^2}{2}$ **(D)** π^2

22. Let *f* and *g* be twice differential even functions on (–2, 2) such that $f\left(\frac{1}{4}\right)=0, f\left(\frac{1}{2}\right)=0, f(1)=1$ and $g\left(\frac{3}{4}\right)=0, g(1)=2$. Then, the minimum number of solutions of $f(x)\,g''(x)+f'(x)\,g'(x)=0$ in (–2, 2) is equal to ______.

23. Let $f:R\to R$ be a function defined by:

$$f(x)\begin{cases} \max\left\{t^3-3t\right\}t\le x & ; \quad x\le 2 \\ x^2+2x-6 & ; \quad 2<x<3 \\ \left[x-3\right]+9 & ; \quad 3\le x\le 5 \\ 2x+1 & ; \quad x>5 \end{cases}$$

Where [*t*] is the greatest integer less than or equal to *t*. Let *m* be the number of points where *f* is not differentiable and $I=\int_{-2}^{2}f(x)dx$. Then the ordered pair (*m*, *I*) is equal to:

(A) $\left(3,\frac{27}{4}\right)$ **(B)** $\left(3,\frac{23}{4}\right)$ **(C)** $\left(4,\frac{27}{4}\right)$ **(D)** $\left(4,\frac{23}{4}\right)$

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Differential Calculus-2	Class - XII | Mathematics

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1. The function $f(x)=\dfrac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$:

(A) increases in $\left(-\infty,\dfrac{1}{2}\right]$ **(B)** decreases in $\left[\dfrac{1}{2},\infty\right)$

(C) decreases in $\left(-\infty,\dfrac{1}{2}\right]$ **(D)** increases in $\left[\dfrac{1}{2},\infty\right)$

2. If the tangent to the curve $y=x^3$ at the point $P(t,t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is:

(A) $-t^3$ **(B)** $2t^3$ **(C)** 0 **(D)** $-2t^3$

3. If the curve $y=ax^2+bx+c, x\in R$, passes through the point (1,2) and the tangent line to this curve at origin is $y=x$, then the possible values of a, b, c, are:

(A) $a=\dfrac{1}{2}, b=\dfrac{1}{2}, c=1$ **(B)** $a=-1, b=1, c=1$

(C) $a=1, b=1, c=0$ **(D)** $a=1, b=0, c=1$

4. If Rolle's theorem holds for the function $f(x)=x^3-ax^2+bx-4,\ x\in[1,2]$ with $f'\left(\dfrac{4}{3}\right)=0$, then ordered pair (a,b) is equal to:

(A) (5, 8) **(B)** (5, – 8) **(C)** (–5, – 8) **(D)** (–5, 8)

5. Let $f(x)$ be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at $x=-1$ and $x=1$. If $\lim_{x\to 0}\dfrac{f(x)}{x^3}=1$, then $5\cdot f(2)$ is equal to _______.

6. The shortest distance between the line $x-y=1$ and the curve $x^2=2y$ is:

(A) $\dfrac{1}{2}$ **(B)** $\dfrac{1}{2\sqrt{2}}$ **(C)** 0 **(D)** $\dfrac{1}{\sqrt{2}}$

7. If the curves $x=y^4$ and $xy=k$ cut at right angles, then $(4k)^6$ is equal to __________.

8. The maximum slope of the curve $y=\dfrac{1}{2}x^4-5x^3+18x^2-19x$ occur at the point:

(A) (2, 9) **(B)** (0, 0) **(C)** (2, 2) **(D)** $\left(3,\dfrac{21}{2}\right)$

9. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is:

(A) An isosceles triangle with base equal to $2r$.

(B) An equilateral triangle having each of its side of length $\sqrt{3}r$.

(C) A right angle triangle having two of its sides of length $2r$ and r.

(D) An equilateral triangle of height $\dfrac{2r}{3}$.

10. Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4+10x^3+10x^2+10x+10$ lie in the interval $(a, a+1)$. Then, $|a|$ is equal to ________.

11. Consider the function $f: R \to R$ defined by $f(x)=\begin{cases}\left(2-\sin\left(\frac{1}{x}\right)\right)|x| & , x \neq 0 \\ 0 & , x=0\end{cases}$. Then f is:

(A) monotonic on $(-\infty, 0)$ only

(B) monotonic on $(-\infty, 0) \cup (0, \infty)$

(C) monotonic on $(0, \infty)$ only

(D) not monotonic $(-\infty, 0)$ and $(0, \infty)$

12. Let $\alpha \in R$ be such that the function $f(x)=\begin{cases}\frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \neq 0 \\ \alpha, & x=0\end{cases}$ is continuous at $x=0$,

where $\{x\}=x-[x]$, $[x]$ is the greatest integer less than or equal to x. Then:

(A) $\alpha=\frac{\pi}{\sqrt{2}}$

(B) $\alpha=\frac{\pi}{4}$

(C) no such α exists

(D) $\alpha=0$

13. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x+\log_{10}\cos x=-1$ and $\log_{10}(\sin x+\cos x)=\frac{1}{2}(\log_{10} n-1), n>0$, then the value of n is equal to:

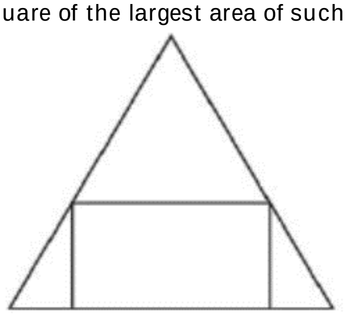
(A) 20 **(B)** 16 **(C)** 12 **(D)** 9

14. The range of $a \in R$ for which the function

$f(x)=(4a-3)(x+\log_e 5)+2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right), x \neq 2n\pi, n \in N$ has critical points, is:

(A) $\left[-\frac{4}{3}, 2\right]$ **(B)** $(-3, 1)$ **(C)** $[1, \infty)$ **(D)** $(-\infty, -1]$

15. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is ______.



16. Let $f(x)=3\sin^4 x+10\sin^3 x+6\sin^2 x-3, x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then, f is:

(A) decreasing in $\left(-\frac{\pi}{6}, 0\right)$

(B) increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

(C) decreasing in $\left(0, \frac{\pi}{2}\right)$

(D) increasing in $\left(-\frac{\pi}{6}, 0\right)$

17. Let $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x)=\begin{cases}-\frac{4}{3}x^3+2x^2+3x, & x>0 \\ 3xe^x & , x \leq 0\end{cases}$$

Then f is increasing function in the interval.

(A) $\left(-1, \frac{3}{2}\right)$ **(B)** $(-3, -1)$ **(C)** $(0, 2)$ **(D)** $\left(-\frac{1}{2}, 2\right)$

18. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15, x \in R$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15, x \in R$ has a:

(A) local maximum at $x = -\frac{3}{4}$ **(B)** local minimum at $x = -\frac{3}{4}$

(C) local minimum at $x = \frac{3}{4}$ **(D)** local maximum at $x = \frac{3}{4}$

19. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} \dfrac{\lambda\left|x^2 - 5x + 6\right|}{\mu(5x - x^2 - 6)} & ,x < 2 \\ e^{\frac{\tan(x-2)}{x-[x]}} & ,x > 2 \\ \mu & ,x = 2 \end{cases}$$

Where $[x]$ is the greatest integer less than or equal to x. If f is continuous at $x = 2$, then $\lambda + \mu$ is equal to:

(A) 1 **(B)** $e(-e+1)$ **(C)** $2e - 1$ **(D)** $e(e-2)$

20. Let $f(x)$ be a cubic polynomial with $f(1) = -10$, $f(-1) = 6$, and has a local minima at $x = 1$, and $f'(x)$ has a local minima at $x = -1$. Then $f(3)$ is equal to _______.

21. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is ___________.

22. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is:

(A) $\frac{10}{3 + 2\sqrt{3}}$ **(B)** $\frac{5}{2 + \sqrt{3}}$ **(C)** $\frac{5}{3 + \sqrt{3}}$ **(D)** $\frac{10}{2 + 3\sqrt{3}}$

23. The local maximum value of the function $f(x) = \left(\frac{2}{x}\right)^{x^2}, x > 0$, is:

(A) $(e)^{\frac{2}{e}}$ **(B)** $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$ **(C)** 1 **(D)** $(2\sqrt{e})^{\frac{1}{e}}$

24. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi} + 1\right)k$ is equal to _______.

25. A box open from top is made from a rectangular sheet of dimension $a \times b$ by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to:

(A) $\frac{a + b + \sqrt{a^2 + b^2 - ab}}{6}$ **(B)** $\frac{a + b - \sqrt{a^2 + b^2 - ab}}{12}$

(C) $\frac{a + b - \sqrt{a^2 + b^2 - ab}}{6}$ **(D)** $\frac{a + b - \sqrt{a^2 + b^2 + ab}}{6}$

26. Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$. Then the value of $\tan(M - m)$ is equal to:

(A) $3 - 2\sqrt{2}$ **(B)** $2 - \sqrt{3}$ **(C)** $3 + 2\sqrt{2}$ **(D)** $2 + \sqrt{3}$

27. Let $f : R \to R$ be defined as:

$$f(x) = \begin{cases} -55x, & if \quad x < -5 \\ 2x^3 - 3x^2 - 120x, & if \quad -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336 & if \quad x > 4, \end{cases}$$

Let $A = \{x \in R : f$ is increasing$\}$. Then A is equal to:

(A) $(-\infty, -5) \cup (-4, \infty)$ **(B)** $(-5, -4) \cup (4, \infty)$ **(C)** $(-5, \infty)$ **(D)** $(-\infty, -5) \cup (4, \infty)$

28. Let f be a real valued function, defined on $R - \{-1, 1\}$ and given by $f(x) = 3\log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}$. Then in which of the following intervals, function $f(x)$ is increasing ?

(A) $(-\infty, \infty) - \{-1, 1\}$ **(B)** $\left(-\infty, \frac{1}{2}\right] - \{-1\}$

(C) $(-\infty, -1) \cup \left(\left[\frac{1}{2}, \infty\right) - \{1\}\right)$ **(D)** $\left(-1, \frac{1}{2}\right]$

29. If the normal to the curve $y(x) = \int_0^x (2t^2 - 15t + 10)\, dt$ at a point (a, b) is parallel to the line $x + 3y = -5, a > 1,$ then the value of $|a + 6b|$ is equal to _______.

30. Let $f : (a, b) \to R$ be twice differentiable function such that $f(x) = \int_a^x g(t)dt$ for a differentiable function $g(x)$. If $f(x) = 0$ has exactly five distinct roots in (a, b) then $g(x)g'(x) = 0$ has at least:

(A) Twelve roots in (a, b) **(B)** Seven roots in (a, b)

(C) three roots in (a, b) **(D)** five roots in (a, b)

31. Let a function $g : [0, 4] \to R$ be defined as $g(x) = \begin{cases} \max\limits_{0 \le t \le x} \{t^3 - 6t^2 + 9t - 3\}, & 0 \le x \le 3 \\ 4 - x & , \quad 3 < x \le 4 \end{cases}$, then the number of points in the interval (0, 4) where $g(x)$ is NOT differentiable, is ___________.

32. The sum of all the local minimum values of the twice differentiable function $f : R \to R$ defined by $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ is:

(A) 5 **(B)** –22 **(C)** –27 **(D)** 0

33. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If $f(0) = 0$, $f(1) = 1$ *and* $f(2) = 2$, then:

(A) $f''(x) > 0$ *for all* $x \in (0, 2)$ **(B)** $f''(x) = 0$ *for all* $x \in (0, 2)$

(C) $f'(x) = 0$ *for some* $x \in [0, 2]$ **(D)** $f''(x) = 0$ *for some* $x \in (0, 2)$

34. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is/are:

(A) 2 **(B)** 1 **(C)** 0 **(D)** 4

JEE Advanced 2021

1. Let $f : R \to R$ be defined by $f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$

Then which of the following statements is (are) TRUE?

(A) f is decreasing in the interval (–2, –1)

(B) f is increasing in the interval (1, 2)

(C) f is onto

(D) Range of f is $\left[-\frac{3}{2}, 2\right]$

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Differential Calculus-2	Class - XII | Mathematics

JEE Main 2022

1. Let $f(x)=\begin{cases} x^3-x^2+10x-7, & x\le 1 \\ -2x+\log_2\left(b^2-4\right), & x>1 \end{cases}$. Then the set of all values of b, for which $f(x)$ has maximum value at $x=1$, is:

(A) $(-6,-2)$ **(B)** $(2, 6)$

(C) $[-6,-2)\cup(2,6]$ **(D)** $\left[-\sqrt{6},-2\right)\cup\left(2,\sqrt{6}\right]$

2. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1}\frac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is _______.

3. Let P and Q be any points on the curves $(x-1)^2+(y+1)^2=1$ and $y=x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval:

(A) $\left(0,\frac{1}{4}\right)$ **(B)** $\left(\frac{1}{2},\frac{3}{4}\right)$ **(C)** $\left(\frac{1}{4},\frac{1}{2}\right)$ **(D)** $\left(\frac{3}{4},1\right)$

4. Let $f(x)=3^{(x^2-2)^3+4}, x\in R$. Then which of the following statements are true?

$P: x=0$ is a point of local minima of f

$Q: x=\sqrt{2}$ is a point of inflection of f

$R: f'$ is increasing for $x>\sqrt{2}$

(A) Only P and Q **(B)** Only P and R **(C)** Only Q and R **(D)** All P, Q and R

5. If the tangent to the curve $y=x^3-x^2+x$ at the point (a,b) is also tangent to the curve $y=5x^2+2x-25$ at the point $(2,-1)$, then $|2a+9b|$ is equal to ________.

6. The slope of normal at any point (x,y), $x>0$, $y>0$ on the curve $y=y(x)$ is given by $\frac{x^2}{xy-x^2y^2-1}$. If the curve passes through the point (1, 1), then $e\cdot y(e)$ is equal to:

(A) $\frac{1-\tan(1)}{1+\tan(1)}$ **(B)** $\tan(1)$ **(C)** 1 **(D)** $\frac{1+\tan(1)}{1-\tan(1)}$

7. Let λ^* be the largest value of λ for which the function $f_\lambda(x)=4\lambda x^3-36\lambda x^2+36x+48$ is increasing for all $x\in R$. Then $f_{\lambda^*}(1)+f_{\lambda^*}(-1)$ is equal to:

(A) 36 **(B)** 48 **(C)** 64 **(D)** 72

8. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds, it becomes 7 units, then its radius after 9 seconds is:

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12

9. For the function

$f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5,\ x > 1$, which one of the following is NOT correct?

(A) f is increasing in (1, 2) and decreasing in $(2, \infty)$

(B) $f(x) = -1$ has exactly two solutions

(C) $f'(e) - f''(2) < 0$

(D) $f(x) = 0$ has a root in the interval $(e, e+1)$

10. If the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve:

(A) $x^2 + \frac{y^2}{81} = 2$ **(B)** $\frac{y^2}{9} - x^2 = 8$ **(C)** $y = 4x^2 + 5$ **(D)** $\frac{x}{3} - y^2 = 2$

11. Water is being filled at the rate of $1 cm^3 / \sec$ in a right circular conical vessel (vertex downwards) of height 35*cm* and diameter 14*cm*. When the height of the water level is 10*cm*, the rate (in $cm^2 / \sec$) at which the wet conical surface are of the vessel increases is:

(A) 5 **(B)** $\frac{\sqrt{21}}{5}$ **(C)** $\frac{\sqrt{26}}{5}$ **(D)** $\frac{\sqrt{26}}{10}$

12. If the angle made by the tangent at the point (x_0, y_0) on the curve

$x = 12(t + \sin t \cos t), y = 12(1 + \sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to:

(A) $6(3 + 2\sqrt{2})$ **(B)** $3(7 + 4\sqrt{3})$ **(C)** 27 **(D)** 48

13. Let $f(x) = \left|(x-1)(x^2 - 2x - 3)\right| + x - 3, x \in \mathbb{R}$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then $m + M$ is equal to _________.

14. Consider a cuboid of sides $2x$, $4x$ and $5x$ and a closed hemisphere of radius r. If the sum of their surface areas is a constant k, then the ratio $x : r$, for which the sum of their volumes is maximum, is:

(A) 2 : 5 **(B)** 19 : 45 **(C)** 3 : 8 **(D)** 19 : 15

15. If $y = y(x)$ is the solution of the differential equation $x\frac{dy}{dx} + 2y = xe^x, y(1) = 0$ then the local maximum value of the function $z(x) = x^2 y(x) - e^x, x \in R$ is:

(A) $1 - e$ **(B)** 0 **(C)** $\frac{1}{2}$ **(D)** $\frac{4}{e} - e$

16. The sum of the absolute minimum and the absolute maximum values of the function $f(x) = \left|3x - x^2 + 2\right| - x$ in the interval [–1, 2] is:

(A) $\frac{\sqrt{17}+3}{2}$ **(B)** $\frac{\sqrt{17}+5}{2}$ **(C)** 5 **(D)** $\frac{9-\sqrt{17}}{2}$

17. Let S be the set of all the natural numbers, for which the line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point $(a,b), ab \neq 0$. Then:

(A) $S=\phi$ **(B)** $n(S)=1$ **(C)** $S=\{2k : k \in N\}$ **(D)** $S=N$

18. If m and n respectively are the number of local maximum and local minimum points of the function $f(x)=\int_0^{x^2}\frac{t^2-5t+4}{2+e^t}dt$, then the ordered pair (m,n) is equal to:

(A) $(3, 2)$ **(B)** $(2, 3)$ **(C)** $(2, 2)$ **(D)** $(3, 4)$

19. Let f be a differentiable function in $\left(0,\frac{\pi}{2}\right)$ If $\int_{\cos x}^{1} t^2 f(t)dt=\sin^3 x+\cos x$, then $\frac{1}{\sqrt{3}}f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to:

(A) $6-9\sqrt{2}$ **(B)** $6-\frac{9}{\sqrt{2}}$ **(C)** $\frac{9}{2}-6\sqrt{2}$ **(D)** $\frac{9}{\sqrt{2}}-6$

20. Let the slope of the tangent to a curve $y=f(x)$ at (x,y) be given by $2\tan x(\cos x-y)$. If the curve passes through the point $\left(\pi/4,0\right)$, then the value of $\int_0^{\pi/2} y\,dx$ is equal to:

(A) $\left(2-\sqrt{2}\right)+\frac{\pi}{\sqrt{2}}$ **(B)** $2-\frac{\pi}{\sqrt{2}}$ **(C)** $\left(2+\sqrt{2}\right)+\frac{\pi}{\sqrt{2}}$ **(D)** $2+\frac{\pi}{\sqrt{2}}$

21. The number of real solution of the equation $e^{4x}+4e^{3x}-58e^{2x}+4e^x+1=0$ is ______.

22. Let l be a line which is normal to the curve $y=2x^2+x+2$ at a point P on the curve. If the point $Q(6, 4)$ lies on the line l and O is origin, then the area of the triangle OPQ is equal to ________.

23. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=(x-3)^{n_1}(x-5)^{n_2}, n_1, n_2 \in \mathbb{N}$. Then, which of the following is <u>NOT</u> true?

(A) For $n_1=3, n_2=4$, there exists $\alpha\in(3,5)$ where f attains local maxima

(B) For $n_1=4, n_2=3$, there exist $\alpha\in(3,5)$ where f attains local minima

(C) For $n_1=3, n_2=5$, there exists $\alpha\in(3,5)$ where f attains local maxima

(D) For $n_1=4, n_2=6$, there exists $\alpha\in(3,5)$ where f attains local maxima

24. A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is:

(A) $\frac{22}{9+4\sqrt{3}}$ **(B)** $\frac{66}{9+4\sqrt{3}}$ **(C)** $\frac{22}{4+9\sqrt{3}}$ **(D)** $\frac{66}{4+9\sqrt{3}}$

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Integral Calculus-1	Class - XII | Mathematics

JEE Main 2021

1. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to:

(A) $(-1, 3)$ **(B)** $(1, 3)$ **(C)** $(3, 1)$ **(D)** $(1, -3)$

2. The value of the integral $\int \frac{\sin\theta \cdot \sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$ is :

(Where c is constant of integration)

(A) $\frac{1}{18}[9 - 2\sin^2\theta - 3\sin^4\theta - 6\sin^2\theta]^{3/2} + c$

(B) $\frac{1}{18}[11 - 18\sin^2\theta - 9\sin^4\theta - 2\sin^6\theta]^{3/2} + c$

(C) $\frac{1}{18}[9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta]^{3/2} + c$

(D) $\frac{1}{18}[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta]^{3/2} + c$

3. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx, x > 0,$ is equal to:

(where c is a constant of integration)

(A) $4\log_e |x^2 + 5x - 7| + c$ **(B)** $\frac{1}{4}\log_e |x^2 + 5x - 7| + c$

(C) $\log_e |x^2 + 5x - 7| + c$ **(D)** $\log_e \sqrt{x^2 + 5x - 7} + c$

4. For real numbers α, β, γ and δ, if

$$\int \frac{(x^2 - 1) + \tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4 + 3x^2 + 1)\tan^{-1}\left(\frac{x^2+1}{x}\right)} dx = \alpha \log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right) + \beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right) + \delta\tan^{-1}\left(\frac{x^2+1}{x}\right) + C$$ where

C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to _______ .

5. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2 + 5}}{\sqrt{4x^2 - 4x + 6}} dx$ is equal to:

(where c is a constant of integration)

(A) $\frac{1}{2}\sin\sqrt{(2x-1)^2 + 5} + c$ **(B)** $\frac{1}{2}\cos\sqrt{(2x+1)^2 + 5} + c$

(C) $\frac{1}{2}\sin\sqrt{(2x+1)^2 + 5} + c$ **(D)** $\frac{1}{2}\cos\sqrt{(2x-1)^2 + 5} + c$

6. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx, (x \geq 0), f(0) = 0$ and $f(1) = \frac{1}{k}$, then the value of K is __________ .

7. If $\int \frac{dx}{(x^2+x+1)^2} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C, x > 0$ where C is the constant of integration, then the value of $9(\sqrt{3}a+b)$ is equal to ____________ .

8. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to : (Where C is a constant of integration)

(A) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+C$ **(B)** $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$ **(C)** $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+C$ **(D)** $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+C$

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Paragraph for Question 1 and 2

Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to \mathbb{R}, i = 1, 2,$ and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R$ be functions such that

$g_1(x) = 1, g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$, for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$

Define $S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x) \cdot g_i(x)dx, \quad i = 1, 2$

1. The value of $\frac{16S_1}{\pi}$ is ______.

2. The value of $\frac{48S_2}{\pi^2}$ is _____ .

Paragraph for Question 3 & 4

Let $\psi_1 : [0, \infty) \to \mathbb{R}, \psi_2 : [0, \infty) \to \mathbb{R}, f : [0, \infty) \to R$ and $g : [0, \infty) \to R$ be functions such that $f(0) = g(0) = 0,$

$$\psi_1(x) = e^{-x} + x, \qquad x \geq 0,$$

$$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, \qquad x \geq 0,$$

$$f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2} dt, \qquad x > 0$$

and

$$g(x) = \int_0^{x^2} \sqrt{t}e^{-t} dt, \qquad x > 0$$

3. Which of the following statements is **TRUE**?

(A) $f(\sqrt{\ln 3}) + g(\sqrt{\ln 3}) = \frac{1}{3}$

(B) For every $x > 1$, there exists an $\alpha \in (1, x)$ such that $\psi_1(x) = 1 + \alpha x$

(C) For every $x > 0$, there exists a $\beta \in (0, x)$ such that $\psi_2(x) = 2x(\psi_1(\beta) - 1)$

(D) f is an increasing function on the interval $\left[0, \frac{3}{2}\right]$

4. Which of the following statements is **TRUE**?

(A) $\psi_1(x) \leq 1$, for all $x > 0$

(B) $\psi_2(x) \leq 0$, for all $x > 0$

(C) $f(x) \geq 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all $x \in \left(0, \frac{1}{2}\right)$

(D) $g(x) \leq \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

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Integral Calculus-1	Class - XII | Mathematics

JEE Main 2022

1. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to:

(A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$ **(B)** $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(C) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$ **(D)** $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

2. The integral $\int_0^{\frac{\pi}{2}} \frac{1}{3+2\sin x+\cos x}dx$ is equal to:

(A) $\tan^{-1}(2)$ **(B)** $\tan^{-1}(2)-\frac{\pi}{4}$

(C) $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$ **(D)** $\frac{1}{2}$

3. $\lim_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\frac{n^2}{(n^2+9)(n+3)}+\ldots+\frac{n^2}{(n^2+n^2)(n+n)}\right)$ is equal to:

(A) $\frac{\pi}{8}+\frac{1}{4}\log_e 2$ **(B)** $\frac{\pi}{4}+\frac{1}{8}\log_e 2$ **(C)** $\frac{\pi}{4}-\frac{1}{8}\log_e 2$ **(D)** $\frac{\pi}{8}+\log_e\sqrt{2}$

4. Let $g:(0,\infty)\to R$ be a differentiable function such that

$\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{x\,g(x)}{e^x+1}+c$ for all $x>0$, where c is an arbitrary constant. Then:

(A) g is decreasing in $\left(0,\frac{\pi}{4}\right)$ **(B)** g' is increasing in $\left(0,\frac{\pi}{4}\right)$

(C) $g+g'$ is increasing in $\left(0,\frac{\pi}{2}\right)$ **(D)** $g-g'$ is increasing in $\left(0,\frac{\pi}{2}\right)$

5. $\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to:

(A) $\frac{1}{3}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{12}$

6. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x)+c$, $g(1)=0$, then $g\left(\frac{1}{2}\right)$ is equal to:

(A) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+\frac{\pi}{3}$

(B) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)+\frac{\pi}{3}$

(C) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)-\frac{\pi}{3}$

(D) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)-\frac{\pi}{6}$

7. The integral $\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$ is equal to _______.

8. $\lim\limits_{x\to\frac{1}{\sqrt{2}}}\frac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)}$ is equal to:

(A) $\sqrt{2}$ **(B)** $-\sqrt{2}$ **(C)** $\frac{1}{\sqrt{2}}$ **(D)** $-\frac{1}{\sqrt{2}}$

9. The integral $\int\limits_0^1 \frac{1}{7^{\left[1/x\right]}}dx$, where [.] denotes the greatest integer function, is equal to:

(A) $1+6\log_e\left(\frac{6}{7}\right)$ **(B)** $1-6\log_e\left(\frac{6}{7}\right)$ **(C)** $\log_e\left(\frac{7}{6}\right)$ **(D)** $1-7\log_e\left(\frac{6}{7}\right)$

10. Let a be an integer such that $\lim\limits_{x\to 7}\frac{18-\left[1-x\right]}{\left[x-3a\right]}$ exits, where $\left[t\right]$ is greatest integer $\le t$. Then a is equal to:

(A) –6 **(B)** –2 **(C)** 2 **(D)** 6

11. If $\int\frac{\left(x^2+1\right)e^x}{\left(x+1\right)^2}dx = f\left(x\right)e^x+C$, where C is a constant, then $\frac{d^3f}{dx^3}$ at $x=1$ is equal to:

(A) $\frac{-3}{4}$ **(B)** $\frac{3}{4}$ **(C)** $-\frac{3}{2}$ **(D)** $\frac{3}{2}$

12. Let f be a real valued continuous function on [0, 1] and $f(x)=x+\int\limits_0^1(x-t)\,f(t)dt$. Then, which of the following points (x, y) lies on the curve $y=f(x)$?

(A) (2, 4) **(B)** (1, 2) **(C)** (4, 17) **(D)** (6, 8)

13. For $I\left(x\right)=\int\frac{\sec^2 x-2022}{\sin^{2022}x}dx$, if $I\left(\frac{\pi}{4}\right)=2^{1011}$, then:

(A) $3^{1010}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(B) $3^{1010}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

(C) $3^{1011}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(D) $3^{1011}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

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Integral Calculus-2	Class - XII | Mathematics

JEE Main 2021

1. Let $[t]$ denote the greatest integer $\le t$. Then the value of $8.\int_{-1/2}^{1}([2x]+|x|)dx$ is __________.

2. Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt, 0 \le x \le 1$ and $f(0)=0$, then $\lim_{x\to 0}\frac{1}{x^2}\int_0^x f(t)\,dt$:

(A) does not exist **(B)** equals $\frac{1}{2}$ **(C)** equals 0 **(D)** equals 1

3. The area, enclosed by the curves $y=\sin x+\cos x$ and $y=|\cos x-\sin x|$ and the lines $x=0, x=\frac{\pi}{2}$, is:

(A) $2\sqrt{2}(\sqrt{2}-1)$ **(B)** $4(\sqrt{2}-1)$ **(C)** $2\sqrt{2}(\sqrt{2}+1)$ **(D)** $2(\sqrt{2}+1)$

4. If $f: R\to R$ is given by $f(x)=x+1$, then the value of $\lim_{n\to\infty}\frac{1}{4}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+\ldots+f\left(\frac{5(n-1)}{n}\right)\right]$, is:

(A) $\frac{3}{2}$ **(B)** $\frac{5}{2}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{2}$

5. If $\int_{-a}^{a}(|x|+|x-2|)dx=22, (a>2)$ and $[x]$ denotes the greatest integer $\le x$, then $\int_{a}^{-a}(x+[x])dx$ is equal to _______.

6. The value of the integral, $\int_1^3 [x^2-2x-2]dx$, when $[x]$ denotes the greatest integer less than or equal to x, is:

(A) -4 **(B)** $-\sqrt{2}-\sqrt{3}-1$ **(C)** $-\sqrt{2}-\sqrt{3}+1$ **(D)** -5

7. The area of the region : $R=\{(x.\ y): 5x^2 \le y \le 2x^2+9\}$ is:

(A) $9\sqrt{3}$ square units **(B)** $11\sqrt{3}$ square units
(C) $6\sqrt{3}$ square units **(D)** $12\sqrt{3}$ square units

8. Let $f(x)$ be a differentiable function defined on [0,2] such that $f'(x)=f'(2-x)$ for all $x\in(0,2), f(0)=1 and\ f(2)=e^2$. Then the value of $\int_0^2 f(x)\,dx$ is:

(A) $2(1+e^2)$ **(B)** $2(1-e^2)$ **(C)** $1+e^2$ **(D)** $1-e^2$

9. The value of $\int_{-1}^{1} x^2 e^{[x^3]}dx$, where $[t]$ denotes the greatest integer $\le t$, is :

(A) $\frac{1}{3e}$ **(B)** $\frac{e-1}{3e}$ **(C)** $\frac{e+1}{3}$ **(D)** $\frac{e+1}{3e}$

10. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to_______.

11. The value of $\int_{-2}^{2} |3x^2 - 3x - 6|\, dx$ __________ .

12. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is :

(A) 2π **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{2}$ **(D)** 4π

13. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]} dx$, where $[x]$ is the greatest integer $\le x$, is :

(A) $100(e-1)$ **(B)** $100e$ **(C)** $100(1+e)$ **(D)** $100(1-e)$

14. The area bounded by the lines $y = \left| |x-1| - 2 \right|$ is _______.

15. The value of the integral $\int_0^{\pi} |\sin 2x|\, dx$ is _______.

16. For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to :

(A) 0 **(B)** 1 **(C)** -1 **(D)** $\frac{1}{2}$

17. Let A_1 be the area of the region bounded by the curves $y = \sin x, y = \cos x$ and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y = \sin x, y = \cos x$, x-axis and $x = \frac{\pi}{2}$ in the first quadrant. Then,

(A) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$ **(B)** $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

(C) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$ **(D)** $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

18. If $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1} dx$, for $m, n \ge 1$, and $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \alpha I_{m,n}, \alpha \in R$, then α equals ________ .

19. In the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}} dx = \alpha e^{-1} + \beta^{-1/2} + \gamma$, where α, β, γ are integer and $[x]$ denotes the greatest integer less than or equal to x, then the value of $\alpha + \beta + \gamma$ is equal to :

(A) 20 **(B)** 0 **(C)** 25 **(D)** 10

20. Let $I_n = \int_1^e x^{19}(\log|x|)^n dx$, where $n \in N$. If $(20)I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ equals to _______.

21. Let $f : [-3, 1] \to R$ be given as $f(x) = \begin{cases} \min\{(x+6), x^2\} & , \ -3 \le x \le 0 \\ \max\{\sqrt{x}, x^2\} & , \ 0 \le x \le 1 \end{cases}$

If the area bounded by $y = f(x)$ and x-axis A, then the value of $6A$ is equal to _______.

22. Let $f : R \to R$ be defined as $f(x) = e^{-x} \sin x$. If $F : [0, 1] \to R$ is a differentiable function such that $F(x) = \int_0^x f(t)\, dt$, the value of $\int_0^1 (F'(x) + f(x)) e^x dx$ lies in the interval.

(A) $\left[\frac{330}{360}, \frac{331}{360}\right]$ **(B)** $\left[\frac{327}{360}, \frac{329}{360}\right]$ **(C)** $\left[\frac{331}{360}, \frac{334}{360}\right]$ **(D)** $\left[\frac{335}{360}, \frac{336}{360}\right]$

23. Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2)+g(4-x)=4x^3$ and $g(4-x)+g(x)=0$, then the value of $\int_{-4}^{4} f(x^2)dx$ is ____________ .

24. Consider the integral $I=\int_0^{10}\frac{[x]e^{[x]}}{e^{x-1}}dx$, where $[x]$ denotes the greatest integer less than or equal to x. Then the value of I is equal to:

(A) $9(e+1)$ **(B)** $45(e-1)$ **(C)** $9(e-1)$ **(D)** $45(e+1)$

25. The area bounded by the curve $4y^2=x^2(4-x)(x-2)$ is equal to:

(A) $\frac{3\pi}{8}$ **(B)** $\frac{\pi}{8}$ **(C)** $\frac{\pi}{16}$ **(D)** $\frac{3\pi}{2}$

26. Let $P(x)$ be a real polynomial of degree 3 which vanishes at $x=-3$. Let $P(x)$ have local minima at $x=1$, local maxima at $x=-1$ and $\int_{-1}^{1} P(x)dx=18$, then the sum of all the coefficients of the polynomials $P(x)$ is equal to:

27. If [.] represents the greatest integer function, then the value of $\left|\int_0^{\sqrt{\frac{\pi}{2}}}\left[[x^2]-\cos x\right]dx\right|$ is__________.

28. Let $f:(0,2)\to R$ be defined as $f(x)=\log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right)$.

Then, $\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+.....+f(1)\right)$ is equal to ________.

29. Let $g(x)=\int_0^x f(t)\,dt$, where f is continuous function in [0, 3] such that $\frac{1}{3}\le f(t)\le 1$ for all $t\in[0,1]$ and $0\le f(t)\le\frac{1}{2}$ for all $t\in(1,3]$. The largest possible interval in which $g(3)$ lies is:

(A) $\left[-1,-\frac{1}{2}\right]$ **(B)** $\left[-\frac{3}{2},-1\right]$ **(C)** $\left[\frac{1}{3},2\right]$ **(D)** $[1,3]$

30. Let $f:R\to R$ be a continuous function such that $f(x)+f(x+1)=2$, for all $x\in R$. If $I_1=\int_0^8 f(x)\,dx$ and $I_2=\int_{-1}^{3} f(x)\,dx$, then the value of I_1+2I_2 is equal to ______.

31. If $\int_0^{\pi}(\sin^3 x)e^{-\sin^2 x}dx=\alpha-\frac{\beta}{e}\int_0^1\sqrt{t}\,e^t dt$, then $\alpha+\beta$ is equal to ___________.

32. The area of the region bounded by $y-x=2$ and $x^2=y$ is equal to :

(A) $\frac{16}{3}$ **(B)** $\frac{9}{2}$ **(C)** $\frac{4}{3}$ **(D)** $\frac{2}{3}$

33. Let $F:[3,5]\to R$ be a twice differentiable function on (3, 5) such that $F(x)=e^{-x}\int_3^x(3t^2+2t+4F'(t))dt$.

If $F'(4)=\frac{\alpha e^{\beta}-224}{(e^{\beta}-4)^2}$, then $\alpha+\beta$ is equal to ________.

34. Let the domain of the function $f(x) = \log_4\left(\log_5\left(\log_3\left(18x - x^2 - 77\right)\right)\right)$ be (a, b). Then the value of the integral $\int_a^b \frac{\sin^3 x}{(\sin^3 x + \sin^3(a + b - x))}dx$ is equal to ________.

35. The value of the definite integral $\int\limits_{-\pi/4}^{\pi/2} \frac{dx}{(1+e^{x\cos x})(\sin^4 x + \cos^4 x)}$ is equal to :

(A) $-\frac{\pi}{2}$ **(B)** $-\frac{\pi}{4}$ **(C)** $\frac{\pi}{2\sqrt{2}}$ **(D)** $\frac{\pi}{\sqrt{2}}$

36. If the area of the bounded region $R = \left\{(x, y) : \max\{0, \log_e x\} \le y \le 2^x, \frac{1}{2} \le x \le 2\right\}$ is, $\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$, then the value of $(\alpha + \beta - 2\gamma)^2$ is equal to :

(A) 2 **(B)** 1 **(C)** 8 **(D)** 4

37. The value of $\lim\limits_{n \to \infty} \frac{1}{n}\sum\limits_{j=1}^{n} \frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to :

(A) $2 - \log_e\left(\frac{2}{3}\right)$ **(B)** $5 + \log_e\left(\frac{3}{2}\right)$ **(C)** $1 + 2\log_e\left(\frac{3}{2}\right)$ **(D)** $3 + 2\log_e\left(\frac{2}{3}\right)$

38. The value of the integral $\int\limits_{-1}^{1} \log\left(x + \sqrt{x^2+1}\right)dx$ is:

(A) 2 **(B)** –1 **(C)** 0 **(D)** 1

39. The area (in sq. units) of the region, given by the set $\{(x, y) \in R \times R \mid x \ge 0, 2x^2 \le y \le 4 - 2x\}$ is:

(A) $\frac{7}{3}$ **(B)** $\frac{8}{3}$ **(C)** $\frac{17}{3}$ **(D)** $\frac{13}{3}$

40. The value of the definite integral

$$\int\limits_{\pi/24}^{5\pi/24} \frac{dx}{1 + \sqrt[3]{\tan 2x}}$$ **is:**

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{18}$ **(C)** $\frac{\pi}{6}$ **(D)** $\frac{\pi}{12}$

41. Let $f : [0, \infty) \to [0, \infty)$ be defined as $f(x) = \int_0^x [y]dy$

Where $[x]$ is the greatest integer less than or equal to x. Which of the following is true?

(A) f is continuous at every point in $[0, \infty)$ and differentiable except at the integer points

(B) f is continuous everywhere except at the integer points in $[0, \infty)$

(C) f is both continuous and differentiable except at the integer points in $[0, \infty)$

(D) f is differentiable at every point in $[0, \infty)$

42. The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0, y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is ______.

43. The value of the integral $\int\limits_{-1}^{1} \log_e\left(\sqrt{1-x} + \sqrt{1+x}\right)dx$ is equal to:

(A) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$ **(B)** $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$

(C) $\log_e 2 + \frac{\pi}{2} - 1$ **(D)** $2\log_e 2 + \frac{\pi}{4} - 1$

44. If $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1+4\pi^2}, \alpha \in \mathbb{R},$

where $[x]$ is the greatest integer less than or equal to x, then the value of α is:

(A) $200\,(1-e^{-1})$ **(B)** $50\,(e-1)$ **(C)** $150\,(e^{-1}-1)$ **(D)** $100\,(1-e)$

45. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e(x^2-44x+484)} dx$ is equal to:

(A) 5 **(B)** 8 **(C)** 6 **(D)** 10

46. If $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \ldots \left(1+\frac{n^2}{n^2}\right)^n$, then $\lim_{n\to\infty}(U_n)^{n^{\frac{-4}{2}}}$ is equal to:

(A) $\frac{4}{e}$ **(B)** $\frac{e^2}{16}$ **(C)** $\frac{16}{e^2}$ **(D)** $\frac{4}{e^2}$

47. Let a and b respectively be the points of local maximum and local minimum of the function

$f(x) = 2x^3 - 3x^2 - 12x.$

If A is the total area of the region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, then 4 A is equal to _________.

48. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right) dx$ is:

(A) $\frac{5\pi}{4}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{3\pi}{2}$ **(D)** $\frac{3\pi}{4}$

49. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}} dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in \mathbb{R}$, $5\alpha + 6\beta = 0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha+\beta)^2$ is equal to:

(A) 25 **(B)** 16 **(C)** 36 **(D)** 100

50. The area of the region $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is ________.

51. The value of $\lim_{n\to\infty} \frac{1}{n} \sum_{r=0}^{2n-1} \frac{n^2}{n^2+4r^2}$ is:

(A) $\frac{1}{2}\tan^{-1}(2)$ **(B)** $\tan^{-1}(4)$ **(C)** $\frac{1}{4}\tan^{-1}(4)$ **(D)** $\frac{1}{2}\tan^{-1}(4)$

52. The value of $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right)^{1/2} dx$ is:

(A) $2\log_e 16$ **(B)** $4\log_e(3+2\sqrt{2})$ **(C)** $\log_e 4$ **(D)** $\log_e 16$

53. The function $f(x)$, that satisfies the condition $f(x) = x + \int_0^{\pi/2} \sin x \cdot \cos y\, f(y)\, dy$, is:

(A) $x + \frac{2}{3}(\pi-2)\sin x$ **(B)** $x + (\pi+2)\sin x$

(C) $x + \frac{\pi}{2}\sin x$ **(D)** $x + (\pi-2)\sin x$

54. If $x\phi(x)=\int_5^x (3t^2-2\phi'(t))dt$, $x>-2$, and $\phi(0)=4$, then $\phi(2)$ is ___________ .

55. If $\int \frac{2e^x+3e^{-x}}{4e^x+7e^{-x}}dx=\frac{1}{14}\left(ux+v\log_e(4e^x+7e^{-x})\right)+C$, where C is a constant of integration, then $u+v$ is equal to _______.

56. The area of the region bounded by the parabola $(y-2)^2=(x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is :

(A) 6 **(B)** 9 **(C)** 10 **(D)** 4

57. The value of the integral $\int_0^1 \frac{\sqrt{x}\,dx}{(1+x)(1+3x)(3+x)}$ is :

(A) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$ **(B)** $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$ **(C)** $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$ **(D)** $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

58. Let a be a positive real number such that $\int_0^a e^{x-[x]}dx=10e-9$

Where $[x]$ is the greatest integer less than or equal to x. Then a is equal to:

(A) $10+\log_e 2$ **(B)** $10-\log_e(1+e)$ **(C)** $10+\log_e 3$ **(D)** $10+\log_e(1+e)$

59. If $I_n=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot^n x\,dx$, then:

(A) $I_2+I_4, I_3+I_5, I_4+I_6$ are in G.P.

(B) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(C) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

(D) $I_2+I_4, (I_3+I_5)^2, I_4+I_6$ are in G.P.

60. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+.....+\frac{n}{(2n-1)^2}\right]$ is equal to:

(A) 1 **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$

61. Let $P(x)=x^2+bx+c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx=1$ and $P(x)$ leaves remainder 5 when it is divided by $(x-2)$. Then the value of $9(b+c)$ is equal to :

(A) 7 **(B)** 15 **(C)** 9 **(D)** 11

62. If [x] denotes the greatest integer less than or equal to x, then the value of the integral

$\int_{-\pi/2}^{\pi/2}\left[[x]-\sin x\right]dx$ is equal to:

(A) 1 **(B)** $-\pi$ **(C)** π **(D)** 0

63. Let $g(t)=\int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$, where $f(x)=\log_e(x+\sqrt{x^2+1})$, $x\in R$. Then which one of the following is correct ?

(A) $g(1)+g(0)=0$ **(B)** $\sqrt{2}\,g(1)=g(0)$ **(C)** $g(1)=g(0)$ **(D)** $g(1)=\sqrt{2}\,g(0)$

JEE Advanced 2021

1. The area of the region $\left\{(x,y):0\le x\le\frac{9}{4},\ 0\le y\le 1, x\ge 3y,\ x+y\ge 2\right\}$ is:

(1) $\frac{11}{32}$ **(2)** $\frac{35}{96}$ **(3)** $\frac{37}{96}$ **(4)** $\frac{13}{32}$

Archive - JEE Main & Advanced

Integral Calculus-2	Class - XII | Mathematics

JEE Main 2022

1. If $n(2n+1)\int_0^1 (1-x^n)^{2n}\,dx = 1177\int_0^1 (1-x^n)^{2n+1}\,dx$, then $n \varepsilon N$ is equal to __________

2. Let a curve $y = y(x)$ pass through the point (3, 3) and the are of the region under this curve, above the x-axis and between the abscissae 3 and $x(>3)$ be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $(\alpha, 6\sqrt{10})$ in the first quadrant, then α is equal to_____.

3. The odd natural number a, such that the area of the region bounded by $y=1, y=3, x=0, \ x=y^a$ is $\frac{364}{3}$, is equal to:

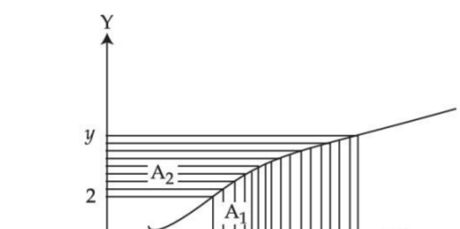
(A) 3 **(B)** 5 **(C)** 7 **(D)** 9

4. Let $f(x) = \min\{[x-1],[x-2],....,[x-10]\}$ Where $[t]$ denotes the greatest integer $\le t$.

Then $\int_0^{10} f(x)dx + \int_0^{10} (f(x))^2 dx + \int_0^{10} |f(x)|dx$ is equal to________.

5. Let f be a differentiable function satisfying $f(x) = \frac{2}{\sqrt{3}}\int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right)d\lambda, x>0$ and $f(1)=\sqrt{3}$. If $y = f(x)$ passes through the point $(\alpha, 6)$, then α is equal to_______.

6. Consider a curve $y = y(x)$ in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line $2x-12y=15$ does NOT pass through the point.



(A) $(6,21)$ **(B)** $(8,9)$ **(C)** $(10,-4)$ **(D)** $(12,-15)$

7. The $\int_0^2 \left(\left|2x^2-3x\right| + \left[x-\frac{1}{2}\right]\right)dx$, where $[t]$ is the greatest integer function, is equal to:

(A) $\frac{7}{6}$ **(B)** $\frac{19}{12}$ **(C)** $\frac{31}{12}$ **(D)** $\frac{3}{2}$

8. The area of the region enclosed by $y \le 4x^2, x^2 \le 9y$ and $y \le 4$, is equal to:

(A) $\frac{40}{3}$ **(B)** $\frac{56}{3}$ **(C)** $\frac{112}{3}$ **(D)** $\frac{80}{3}$

9. Let $f(x) = 2 + |x| - |x-1| + |x+1|, x \in R$.

Consider

$(S1): f'\left(-\frac{3}{2}\right) + f'\left(-\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 2$

$(S2): \int_{-2}^{2} f(x)dx = 12$

Then,

(A) both $(S1)$ and $(S2)$ are correct **(B)** both $(S1)$ and $(S2)$ are wrong

(C) only $(S1)$ is correct **(D)** only $(S2)$ is correct

10. The area bounded by the curves $y = |x^2 - 1|$ and $y = 1$ is:

(A) $\frac{2}{3}(\sqrt{2}+1)$ **(B)** $\frac{4}{3}(\sqrt{2}-1)$ **(C)** $2(\sqrt{2}-1)$ **(D)** $\frac{8}{3}(\sqrt{3}-1)$

11. $\int_0^{20\pi} (|\sin x| + |\cos x|)^2 dx$ is equal to

(A) $10(\pi+4)$ **(B)** $10(\pi+2)$ **(C)** $20(\pi-2)$ **(D)** $20(\pi+2)$

12. If $f(\alpha) = \int_1^{\alpha} \frac{\log_{10} t}{1+t} dt, \alpha > 0$, then $f(e^3) + f(e^{-3})$ is equal to:

(A) 9 **(B)** $\frac{9}{2}$ **(C)** $\frac{9}{\log_e(10)}$ **(D)** $\frac{9}{2\log_e(10)}$

13. The area of the region $\left\{(x,y): |x-1| \le y \le \sqrt{5-x^2}\right\}$ is equal to:

(A) $\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right) - \frac{1}{2}$ **(B)** $\frac{5\pi}{4} - \frac{3}{2}$

(C) $\frac{3\pi}{4} + \frac{3}{2}$ **(D)** $\frac{5\pi}{4} - \frac{1}{2}$

14. The value of the integral $\int_{-\pi/2}^{\pi/2} \frac{dx}{(1+e^x)(\sin^6 x + \cos^6 x)}$ is equal to:

(A) 2π **(B)** 0 **(C)** π **(D)** $\frac{\pi}{2}$

15. The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is _______.

16. Let $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t) dt$. Then the value of $\left|\int_0^{\pi/2} f(\theta)\, d\theta\right|$ is ___________.

17. Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1 and R_2.

If max $\{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to _____________.

18. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is:

(A) $\frac{1}{3}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$

19. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx, n \in \mathbb{N}$, then:

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a A.P. with common difference –2

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference –2

20. The value of $b > 3$ for which $12\int_3^b \frac{1}{\left(x^2 - 1\right)\left(x^2 - 4\right)} dx = \log_e\left(\frac{49}{40}\right)$, is equal to __________.

21. The value of $\int_0^{\pi} \frac{e^{\cos x} \sin x}{(1 + \cos^2 x)(e^{\cos x} + e^{-\cos x})} dx$ is equal to:

(A) $\frac{\pi^2}{4}$ **(B)** $\frac{\pi^2}{2}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{\pi}{2}$

22. The area of the region bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is equal to:

(A) $\frac{32}{3}$ **(B)** $\frac{40}{3}$ **(C)** 16 **(D)** 19

23. The area bounded by the curve $y = |x^2 - 9|$ and the line $y = 3$ is:

(A) $4(2\sqrt{3} + \sqrt{6} - 4)$ **(B)** $4(4\sqrt{3} + \sqrt{6} - 4)$

(C) $8(4\sqrt{3} + 3\sqrt{6} - 9)$ **(D)** $8(4\sqrt{3} + \sqrt{6} - 9)$

24. Let $f(x) = \max\{|x+1|, |x+2|, ..., |x+5|\}$. Then $\int_{-6}^{0} f(x)dx$ is equal to____________.

25. The value of the integral $\frac{48}{\pi^4}\int_0^{\pi}\left(\frac{3\pi x^2}{2} - x^3\right)\frac{\sin x}{1 + \cos^2 x} dx$ is equal to ________.

26. If the area of the region $\left\{(x, y) : x^{\frac{2}{3}} + y^{\frac{2}{3}} \le 1, x + y \ge 0, y \ge 0\right\}$ is A, then $\frac{256A}{\pi}$ is equal to _________.

27. The value of the integral $\int_{-2}^{2} \frac{\left|x^3 + x\right|}{\left(e^{x|x|} + 1\right)} dx$ is equal to:

(A) $5e^2$ **(B)** $3e^{-2}$ **(C)** 4 **(D)** 6

28. Let $A_1 = \left\{(x, y) : |x| \le y^2, |x| + 2y \le 8\right\}$ and $A_2 = \left\{(x, y) : |x| + |y| \le k\right\}$. If $27(Area\ A_1) = 5(Area\ A_2)$, then k is equal to:

29. Let $f: R \to R$ be a continuous function satisfying $f(x) + f(x+k) = n$, for all $x \in R$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x)dx$ and $I_2 = \int_{-k}^{3k} f(x)dx$, then:

(A) $I_1 + 2I_2 = 4nk$ **(B)** $I_1 + 2I_2 = 2nk$

(C) $I_1 + nI_2 = 4n^2k$ **(D)** $I_1 + nI_2 = 6n^2k$

30. The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$ and the x – axis is:

(A) $\frac{9}{4}$ **(B)** $\frac{45}{16}$ **(C)** $\frac{27}{8}$ **(D)** $\frac{63}{16}$

31. Let $[t]$ denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 \left[-8x^2 + 6x - 1\right]dx$ is equal to:

(A) -1 **(B)** $\frac{-5}{4}$ **(C)** $\frac{\sqrt{17}-13}{8}$ **(D)** $\frac{\sqrt{17}-16}{8}$

32. The area of the region $S = \{(x,y) : y^2 \le 8x, y \ge \sqrt{2}x, x \ge 1\}$ is:

(A) $\frac{13\sqrt{2}}{6}$ **(B)** $\frac{11\sqrt{2}}{6}$ **(C)** $\frac{5\sqrt{2}}{6}$ **(D)** $\frac{19\sqrt{2}}{6}$

33. If $\int_0^2 \left(\sqrt{2x} - \sqrt{2x - x^2}\right) dx = \int_0^1 \left(1 - \sqrt{1-y^2} - \frac{y^2}{2}\right) dy + \int_1^2 \left(2 - \frac{y^2}{2}\right) dy + I$ then I equals:

(A) $\int_0^1 \left(1 + \sqrt{1-y^2}\right) dy$ **(B)** $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1-y^2} + 1\right) dy$

(C) $\int_0^1 \left(1 - \sqrt{1-y^2}\right) dy$ **(D)** $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1-y^2} + 1\right) dy$

34. For real numbers a, b ($a > b > 0$), let

Area $\left\{(x,y) : x^2 + y^2 \le a^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \ge 1\right\} = 30\pi$ and $\left\{(x,y) : x^2 + y^2 \ge b^2 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\} = 18\pi$

Then the value of $(a-b)^2$ is equal to ______.

35. The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}x$ that lies outside the triangle formed by $y = \sqrt{2}x, x = 1, y = 2\sqrt{2}$ is equal to:

(A) $\frac{16\sqrt{2}}{6}$ **(B)** $\frac{11\sqrt{2}}{6}$ **(C)** $\frac{13\sqrt{2}}{6}$ **(D)** $\frac{5\sqrt{2}}{6}$

36. $\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right) dx$, Where [t] denotes greatest integer less than or equal to t, is equal to:

(A) -3 **(B)** -2 **(C)** 2 **(D)** 0

Archive - JEE Main & Advanced

Differential Equations	Class - XII | Mathematics

JEE Main 2021

1. Let the curve $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} = 2(x+1)$. If the numerical value of area bounded by the curve $y = y(x)$ and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of $y(1)$ is equal to _______.

2. If a curve $y = f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?

(A) $\frac{62}{5}$ **(B)** $\frac{31}{5}$ **(C)** 5 **(D)** 10

3. The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is :

(A) $\frac{1}{2}\log_e 18$ **(B)** $\log_e 18$ **(C)** $\log_e 9$ **(D)** $2\log_e 18$

4. Let f be a twice differentiable function defined on R such that $f(0)=1$, $f'(0)=2$ and $f'(x) \neq 0$ for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of $f(1)$ lies in the interval:

(A) (3,6) **(B)** (0, 3) **(C)** (6, 9) **(D)** (9, 12)

5. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2 - 4x + y + 8}{x - 2}$, then this curve also passes through the point :

(A) (4, 5) **(B)** (5, 5) **(C)** (5, 4) **(D)** (4, 4)

6. If the curve, $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to ____ .

7. The rate of growth of bacteria in a culture is proportional to the number of bacteria percent and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to :

(A) 16 **(B)** 8 **(C)** 2 **(D)** 4

8. If $y = y(x)$ is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x$, $y(0) = 0$; then $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to _______.

9. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is _______.

10. Let $f(x) = \int_0^x e^t f(t)dt + e^x$ be a differentiable function for all $x \in R$. Then $f(x)$ equals:

(A) $e^{e^x} - 1$ **(B)** $2e^{e^x} - 1$ **(C)** $e^{(e^x - 1)}$ **(D)** $2e^{(e^x - 1)} - 1$

11. Let slope of the tangent line to a curve at any point $P(x, y)$ be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point $(3, y)$ lies on the curve, is:

(A) $\frac{18}{35}$ **(B)** $-\frac{18}{11}$ **(C)** $-\frac{18}{19}$ **(D)** $-\frac{4}{3}$

12. Let $y = y(x)$ be the solution of the differentiable equation

$\cos x(3\sin x + \cos x + 3)dy = (1 + y\sin x(3\sin x + \cos x + 3))dx,\ 0 \le x \le \frac{\pi}{2},\ y(0) = 0.$

Then, $y\left(\frac{\pi}{3}\right)$ is equal to :

(A) $2\log_e\left(\frac{3\sqrt{3} - 8}{4}\right)$ **(B)** $2\log_e\left(\frac{\sqrt{3} + 7}{2}\right)$

(C) $2\log_e\left(\frac{2\sqrt{3} + 10}{11}\right)$ **(D)** $2\log_e\left(\frac{2\sqrt{3} + 9}{6}\right)$

13. In the curve $y = y(x)$ is the solution of the differentiable equation

$2(x^2 + x^{5/4})dy - y(x + x^{1/4})dx = 2x^{9/4}dx,\ x > 0$ which passes through the point $\left(1, 1 - \frac{4}{3}\log_e 2\right)$, then the value of $y(16)$ is equal to :

(A) $4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$ **(B)** $\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

(C) $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$ **(D)** $\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

14. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is:

(A) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$ **(B)** $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

(C) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$ **(D)** $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

15. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x)y = \sin x,\ 0 \le x \le \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

(A) $\log_e 2$ **(B)** $\frac{1}{2}\log_e 2$ **(C)** $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$ **(D)** $\frac{1}{4}\log_e 2$

16. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx}=y^2-x^2, x>0$. Let the curve C_2 be the solution of $\frac{2xy}{x^2-y^2}=\frac{dy}{dx}$. If both the curves pass through (1, 1), then the area enclosed by the curves C_1 and C_2 is equal to :

(A) $\pi+1$ **(B)** $\frac{\pi}{2}-1$ **(C)** $\pi-1$ **(D)** $\frac{\pi}{4}+1$

17. Which of the following is true for $y(x)$ that satisfies the differential equation $\frac{dy}{dx}=xy-1+x-y$; $y(0)=0$:

(A) $y(1)=1$ **(B)** $y(1)=e^{\frac{1}{2}}-1$ **(C)** $y(1)=e^{\frac{1}{2}}-e^{-\frac{1}{2}}$ **(D)** $y(1)=e^{-\frac{1}{2}}-1$

18. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}=(y+1)\left((y+1)e^{x^2/2}-x\right),\ 0<x<2.1$, with $y(2)=0$. Then the value of $\frac{dy}{dx}$ at $x=1$ is equal to:

(A) $\frac{e^{5/2}}{(1+e^2)^2}$ **(B)** $\frac{5e^{1/2}}{(e^2+1)^2}$ **(C)** $\frac{-e^{3/2}}{(e^2+1)^2}$ **(D)** $-\frac{2e^2}{(1+e^2)^2}$

19. If $y=y(x)$ is the solution of the differential equation, $\frac{dy}{dx}+2y\tan x=\sin x, y\left(\frac{\pi}{3}\right)=0$, then the maximum value of the function $y(x)$ over R is equal to :

(A) $\frac{1}{8}$ **(B)** $\frac{1}{2}$ **(C)** 8 **(D)** $-\frac{15}{4}$

20. Let $y=y(x)$ be the solution of the differential equal to $dy=e^{\alpha x+y}dx : \alpha\in \boldsymbol{N}$.
If $y(\log_e 2)=\log_e 2$ and $y(0)=\log_e\left(\frac{1}{2}\right)$, then the value of α is equal to __________.

21. Let $y=y(x)$ be the solution of the differential equation $(x-x^3)dy=(y+yx^2-3x^4)dx, x>2$. If $y(3)=3$, then $y(4)$ is equal to :

(A) 12 **(B)** 8 **(C)** 4 **(D)** 16

22. If $y=y(x), y\in\left[0,\frac{\pi}{2}\right)$ is the solution of the differential equation $\sec y\frac{dy}{dx}-\sin(x+y)-\sin(x-y)=0$, with $y(0)=0$, then $5y'\left(\frac{\pi}{2}\right)$ is equal to _______.

23. Let $y=y(x)$ be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right)=3x+4y$, with $y(0)=0$.
If $y\left(-\frac{2}{3}\log_e 2\right)=\alpha\log_e 2$, then the value of α is equal to :

(A) $\frac{1}{4}$ **(B)** 2 **(C)** $-\frac{1}{2}$ **(D)** $-\frac{1}{4}$

24. Let a curve $y=f(x)$ pass through the point $(2,(\log_e 2)^2)$ and have slope $\frac{2y}{x\log_e x}$ for all positive real value of x. Then the value of $f(e)$ is equal to ______.

25. Let $y = y(x)$ be the solution of the differential equation $xdy = (y + x^3 \cos x)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(A) $\frac{\pi^2}{2}+\frac{\pi}{4}$ **(B)** $\frac{\pi^2}{4}-\frac{\pi}{2}$ **(C)** $\frac{\pi^2}{4}+\frac{\pi}{2}$ **(D)** $\frac{\pi^2}{2}-\frac{\pi}{4}$

26. Let $y = y(x)$ be solution of the following differential equation

$$e^y \frac{dy}{dx} - 2e^y \sin x + \sin x \cos^2 x = 0, \ y\left(\frac{\pi}{2}\right) = 0.$$

If $y(0) = \log_e(\alpha + \beta e^{-2})$, then $4(\alpha+\beta)$ is equal to__________ .

27. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 1 + xe^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$ then, the minimum value of $y(x), x \in (-\sqrt{2}, 2)$ is equal to:

(A) $(1+\sqrt{3}) - \log_e(\sqrt{3}-1)$ **(B)** $(2+\sqrt{3}) + \log_e 2$

(C) $(1-\sqrt{3}) - \log_e(\sqrt{3}-1)$ **(D)** $(2-\sqrt{3}) - \log_e 2$

28. Let $y = y(x)$ be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x+2)\,dy, y(1) = 1$.

If the domain of $y = y(x)$ is an open interval (α, β), then $|\alpha+\beta|$ is equal to ______.

29. Let $y = y(x)$ be the solution of the differential equation $\text{cosec}^2 x\ dy + 2dx = (1 + y\cos 2x)\text{cosec}^2 x\ dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then, the value of $(y(0)+1)^2$ is equal to:

(A) $e^{1/2}$ **(B)** e **(C)** $e^{-1/2}$ **(D)** e^{-1}

30. Let a curve $y = y(x)$ be given by the solution of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}dy$$

If it intersects y-axis at $y = -1$, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then e^{α} is equal to _____.

31. Let $y = y(x)$ satisfies the equation $\frac{dy}{dx} - |A| = 0$, for all $x > 0$, where $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$. If $y(\pi) = \pi + 2$, then the value of $y\left(\frac{\pi}{2}\right)$ is:

(A) $\frac{\pi}{2}+\frac{4}{\pi}$ **(B)** $\frac{3\pi}{2}-\frac{1}{\pi}$ **(C)** $\frac{\pi}{2}-\frac{4}{\pi}$ **(D)** $\frac{\pi}{2}-\frac{1}{\pi}$

32. Let $y = y(x)$ be the solution of the differential equation

$$x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx, -1 \le x \le 1, y\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$

Then the area of the region bounded by the curves $x = 0, x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is:

(A) $\frac{1}{8}(\pi-1)$ **(B)** $\frac{1}{4}(\pi-2)$ **(C)** $\frac{1}{6}(\pi-1)$ **(D)** $\frac{1}{12}(\pi-3)$

33. Let $y = y(x)$ be the solution of the differential equation $e^x \sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right)dy = 0, y(1) = -1$

Then the value of $\left(y(3)\right)^2$ is equal to :

(A) $1-4e^3$ **(B)** $1+4e^6$ **(C)** $1+4e^3$ **(D)** $1-4e^6$

34. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2(y+2\sin x - 5)x - 2\cos x$ such that $y(0) = 7$. Then $y(\pi)$ is equal to :

(A) $2e^{\pi^2}+5$ **(B)** $7e^{\pi^2}+5$ **(C)** $e^{\pi^2}+5$ **(D)** $3e^{\pi^2}+5$

35. Let us consider a curve, $y = f(x)$ passing through the point (–2, 2) and the slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$. Then:

(A) $x^3 + xf(x) + 12 = 0$ **(B)** $x^2 + 2xf(x) + 4 = 0$

(C) $x^3 - 3xf(x) - 4 = 0$ **(D)** $x^2 + 2xf(x) - 12 = 0$

36. Let $y(x)$ be the solution of the differential equation $2x^2 dy + (e^y - 2x)dx = 0$, $x > 0$. If $y(e) = 1$, then $y(1)$ is equal to:

(A) $\log_e 2$ **(B)** $\log_e(2e)$ **(C)** 2 **(D)** 0

37. Let $y = y(x)$ be a solution curve of the differential equation $(y+1)\tan^2 x\, dx + \tan x\, dy + y\, dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim\limits_{x \to 0^+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is:

(A) $\frac{\pi}{4}+1$ **(B)** $\frac{\pi}{4}-1$ **(C)** $-\frac{\pi}{4}$ **(D)** $\frac{\pi}{4}$

38. If $y = y(x)$ is the solution curve of the differential equation $x^2 dy + \left(y - \frac{1}{x}\right)dx = 0$; $x > 0$, and $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to:

(A) $3-e$ **(B)** $3+\frac{1}{\sqrt{e}}$ **(C)** $3+e$ **(D)** $\frac{3}{2}-\frac{1}{\sqrt{e}}$

39. If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, $y(0) = 1$, then $y(1)$ is equal to:

(A) $\log_2(1+e)$ **(B)** $\log_2(2e)$ **(C)** $\log_2(2+e)$ **(D)** $\log_2(1+e^2)$

40. If the solution curve of the differential equation $(2x - 10y^3)dy + ydx = 0$, passes through the points $(0, 1)$ and $(2, \beta)$, then β is a root of the equation :

(A) $2y^5 - y^2 - 2 = 0$ **(B)** $y^5 - y^2 - 1 = 0$

(C) $y^5 - 2y - 2 = 0$ **(D)** $2y^5 - 2y - 1 = 0$

JEE Advanced 2021

1. For any real numbers α and β, let $y_{\alpha,\beta}(x), x \in \mathbb{R}$, be the solution of the differential equation

$$\frac{dy}{dx} + \alpha y = xe^{\beta x}, y(1) = 1.$$

Let $S = \{y_{\alpha,\beta}(x) : \alpha, \beta \in \mathbb{R}\}$. Then which of the following functions belong(s) to the set S?

(A) $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$

(B) $f(x) = -\frac{x^2}{2}e^{-x} + \left(e + \frac{1}{2}\right)e^{-x}$

(C) $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{4}\right)e^{-x}$

(D) $f(x) = \frac{e^x}{2}\left(\frac{1}{2} - x\right) + \left(e + \frac{e^2}{4}\right)e^{-x}$

Question Stem for Question Nos. 2 and 3

Question Stem

Let $f_1 : (0, \infty) \to \mathbb{R}$ and $f_2 : (0, \infty) \to \mathbb{R}$ be defined by

$$f_1(x) = \int_0^x \prod_{j=1}^{21} (t - j)^j \, dt, \quad x > 0 \text{ and } f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450, \quad x > 0,$$

Where, for any positive integer n and real numbers $a_1, a_2, \ldots, a_n, \prod_{i=1}^{n} a_i$ denotes the product of $a_1, a_2, \ldots, a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maximum of function $f_i, i = 1, 2,$ in the interval $(0, \infty)$.

2. The value of $2m_1 + 3n_1 + m_1 n_1$ is ________ .

3. The value of $6m_2 + 4n_2 + 8m_2 n_2$ is _________ .

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Archive - JEE Main & Advanced

Differential Equations	Class - XII | Mathematics

JEE Main 2022

1. If $\frac{dy}{dx}+2y\tan x=\sin x,\ 0<x<\frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right)=0$, then the maximum value of $y(x)$ is:

(A) $\frac{1}{8}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{3}{8}$

2. For the curve $C:\left(x^2+y^2-3\right)+\left(x^2-y^2-1\right)^5=0$, the value of $3y'-y^3y''$, at the point (α,α), $\alpha>0$, on C, is equal to_______.

3. Suppose $y=y(x)$ be the solution curve to the differential equation $\frac{dy}{dx}-y=2-e^{-x}$ such that $\lim_{x\to\infty}y(x)$ is finite. If a and b are respectively the x- and y- intercepts of the tangent to the curve at $x=0$, then the value of $a-4b$ is equal to _______.

4. Let the solution curve $y=f(x)$ of the differential equation $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}},x\in(-1,1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}f(x)dx$ is equal to:

(A) $\frac{\pi}{3}-\frac{1}{4}$ **(B)** $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$ **(C)** $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$ **(D)** $\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

5. Let the solution curve $y=y(x)$ of the differential equation $\left(1+e^{2x}\right)\left(\frac{dy}{dx}+y\right)=1$ pass through the point $\left(0,\frac{\pi}{2}\right)$. Then, $\lim_{x\to\infty}e^xy(x)$ is equal to:

(A) $\frac{\pi}{4}$ **(B)** $\frac{3\pi}{4}$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{3\pi}{2}$

6. If $x=x(y)$ is the solution of the differential equation $y\frac{dx}{dy}=2x+y^3(y+1)e^y$, $x(1)=0$; then $x(e)$ is equal to:

(A) $e^3(e^e-1)$ **(B)** $e^e(e^3-1)$ **(C)** $e^2(e^e+1)$ **(D)** $e^e(e^2-1)$

7. If $y=y(x)$ is the solution of the differential equation $2x^2\frac{dy}{dx}-2xy+3y^2=0$ such that $y(e)=\frac{e}{3}$, then $y(1)$ is equal to:

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{2}$ **(D)** 3

8. Let $y = y(x)$ be the solution of the differential equation $(x+1)y' - y = e^{3x}(x+1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the curve $y = y(x)$ is:

(A) Not a critical point **(B)** A point of local minima

(C) A point of local maxima **(D)** A point of inflection

9. If the solution curve $y = y(x)$ of the differential equation $y^2 dx + (x^2 - xy + y^2)dy = 0$, which passes through the point (1, 1) and intersects the line $y = \sqrt{3}x$ at the point $(\alpha, \sqrt{3}\alpha)$, then value of $\log_e(\sqrt{3}\alpha)$ is equal to:

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi}{12}$ **(D)** $\frac{\pi}{6}$

10. If the solution of the differential equation $\frac{dy}{dx} + e^x(x^2 - 2)y = (x^2 - 2x)(x^2 - 2)e^{2x}$ satisfies $y(0) = 0$, then the value of $y(2)$ is _______.

(A) -1 **(B)** 1 **(C)** 0 **(D)** e

11. Let the solution curve $y = y(x)$ of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the origin. Then $y(2)$ is equal to ____________.

12. Let $S = (0, 2\pi) - \left\{\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\}$. Let $y = y(x), x \in S$, be the solution curve of the differential equation $\frac{dy}{dx} = \frac{1}{1 + \sin 2x}, y\left(\frac{\pi}{4}\right) = \frac{1}{2}$. If the sum of abscissas of all the points of intersection of the curve $y = y(x)$ with the curve $(y = \sqrt{2}\sin x)$ is $\frac{k\pi}{12}$, then k is equal to _____________.

13. If the solution curve of the differential equation $\left(\left(\tan^{-1} y\right) - x\right)dy = \left(1 + y^2\right)dx$ passes through the point $(1, 0)$, then the abscissa of the point on the curve whose ordinate is $\tan(1)$, is:

(A) $2e$ **(B)** $\frac{2}{e}$ **(C)** 2 **(D)** $\frac{1}{e}$

14. If $y(x) = \left(x^x\right)^x, x > 0$, then $\frac{d^2x}{dy^2} + 20$ at $x = 1$ is equal to ______.

15. Let $y = y(x)$ be the solution of the differential equation

$$\left(1 - x^2\right)dy = \left(xy + \left(x^3 + 2\right)\sqrt{1 - x^2}\right)dx, -1 < x < 1, \text{ and } y(0) = 0.$$ If $\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1 - x^2}\, y(x)dx = k$, then k^{-1} is equal to ______.

16. Let $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$, where a, b, c are constants, represent a circle passing through the point $(2, 5)$. Then the shortest distance of the point (11, 6) from this circle is:

(A) 10 **(B)** 8 **(C)** 7 **(D)** 5

17. If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$, $|y| < 2$, then:

(A) $x^2y'' + xy' - 25y = 0$ **(B)** $x^2y'' - xy' - 25y = 0$

(C) $x^2y'' - xy' + 25y = 0$ **(D)** $x^2y'' + xy' + 25y = 0$

18. If $\frac{dy}{dx} + \frac{2^{x-y}\left(2^y - 1\right)}{2^x - 1} = 0, x, y > 0, y(1) = 1,$ then $y(2)$ is equal to:

(A) $2 + \log_2 3$ **(B)** $2 + \log_3 2$ **(C)** $2 - \log_3 2$ **(D)** $2 - \log_2 3$

19. Let $x = x(y)$ be the solution of the differential equation $2ye^{x/y^2}dx + \left(y^2 - 4xe^{x/y^2}\right)dy = 0$ such that $x(1) = 0$. Then, $x(e)$ is equal to:

(A) $e\ \log_e(2)$ **(B)** $-e\ \log_e(2)$ **(C)** $e^2\ \log_e(2)$ **(D)** $-e^2\ \log_e(2)$

20. Let the solution curve $y = y(x)$ of the differential equation

$$\left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right]x\frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right]y$$

Pass through the points (1, 0) and $(2\alpha, \alpha)$, $\alpha > 0$. Then α is equal to:

(A) $\frac{1}{2}\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$ **(B)** $\frac{1}{2}\exp\left(\frac{\pi}{3} + e - 1\right)$

(C) $\exp\left(\frac{\pi}{6} + \sqrt{e} + 1\right)$ **(D)** $2\exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$

21. Let $y = y(x)$ be the solution of the differential equation $x\left(1 - x^2\right)\frac{dy}{dx} + \left(3x^2y - y - 4x^3\right) = 0$, $x > 1$, with $y(2) = -2$. Then $y(3)$ is equal to:

(A) –18 **(B)** –12 **(C)** –6 **(D)** –3

22. If $y = y(x)$ is the solution of the differential equation $(1 + e^{2x})\frac{dy}{dx} + 2(1 + y^2)e^x = 0$ and $y(0) = 0$, then $6\left(y'(0) + (y(\log_e\sqrt{3}))^2\right)$ is equal to:

(A) 2 **(B)** –2 **(C)** –4 **(D)** 1

23. Let $y = y(x)$, $x > 1$, be the solution of the differential equation $(x - 1)\frac{dy}{dx} + 2xy = \frac{1}{x - 1}$, with $y(2) = \frac{1 + e^4}{2e^4}$. If $y(3) = \frac{e^\alpha + 1}{\beta e^\alpha}$, then the value of $\alpha + \beta$ is equal to ______.

24. Let the solutions curve of the differential equation:

$x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$. Then $y(2)$ is equal to:

(A) 15 **(B)** 11 **(C)** 13 **(D)** 17

25. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^4 x - \cos 2x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}$, $0 < x < \frac{\pi}{2}$ with $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$. If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to ________.

26. If the solution curve of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passes through the points $(2,1)$ and $(k+1,2), k>0$, then

(A) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$ **(B)** $\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(C) $2\tan^{-1}\left(\frac{1}{k+1}\right)=\log_e\left(k^2+2k+2\right)$ **(D)** $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(\frac{k^2+1}{k^2}\right)$

27. Let $y=y(x)$ be the solution curve of the differential equation $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{(x+3)}{x+1}, x>-1$, which passes through the point (0, 1). The $y(1)$ is equal to

(A) $\frac{1}{2}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{5}{2}$ **(D)** $\frac{7}{2}$